Curriculum and Task Design

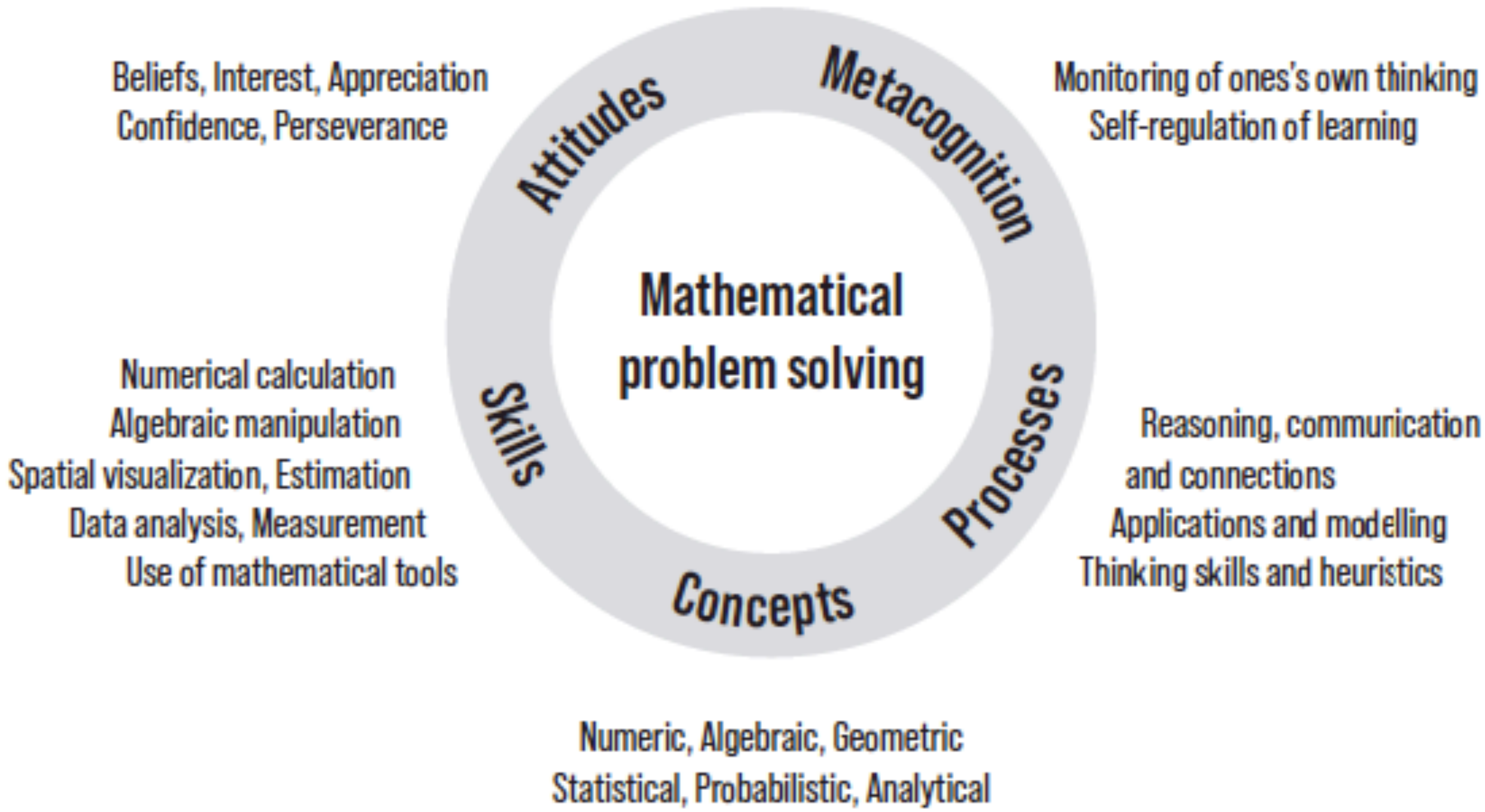
Chris McGrane
PT Maths
Holyrood Secondary
Glasgow

STARTING POINTS MATHS
TASKS AND BLOG

startingpointsmaths.com

@chrismcgrane84

Procedural Fluency, Conceptual Understanding, Mathematical Behaviours



(Singapore Ministry of Education, 2012)

Education Endowment Foundation

The Big Picture | Evidence summaries | Practical Tools | Projects and Evaluation | Support for schools | Covid-19 Resources | News | More

Improving Mathematics in Key Stages 2 and 3

Eight recommendations to improve outcomes in maths for 7-14 year olds

Published 3rd November, 2017

Scroll down to read more

Download the full Guidance Report

Full PDF: 6 MB

Summary of recommendations poster

The main recommendations summarised in a downloadable poster. [30 KB pdf]

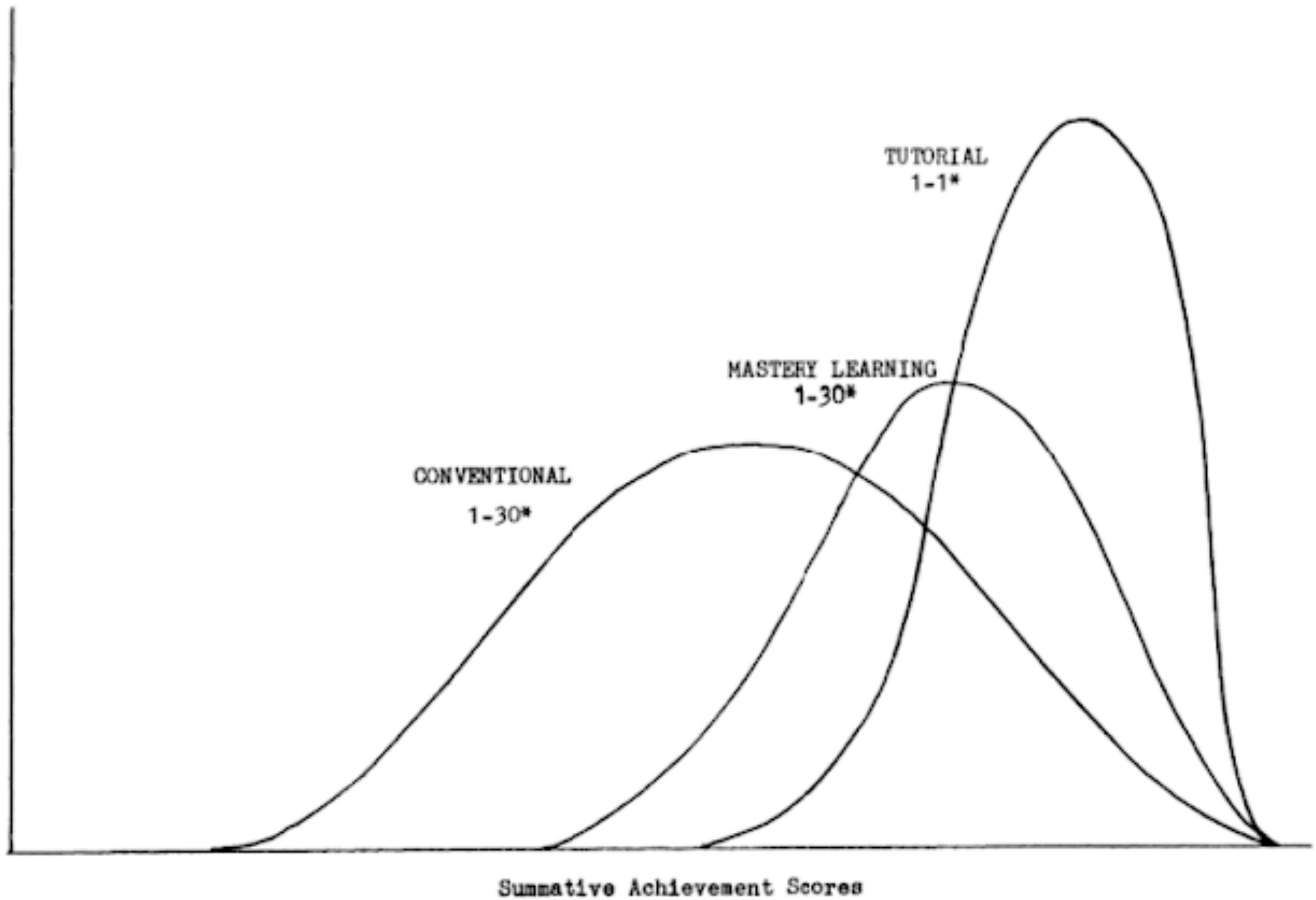
Recommendation 1	Use assessment to build on pupils' existing knowledge and understanding
Recommendation 2	Use manipulatives and representations
Recommendation 3	Teach strategies for solving problems
Recommendation 4	Enable pupils to develop a rich network of mathematical knowledge
Recommendation 5	Develop pupils' independence and motivation
Recommendation 6	Use tasks and resources to challenge and support pupils' mathematics
Recommendation 7	Use structured interventions to provide additional support
Recommendation 8	Support pupils to make a successful transition between primary and secondary school

Observe the lesson through a student's eyes

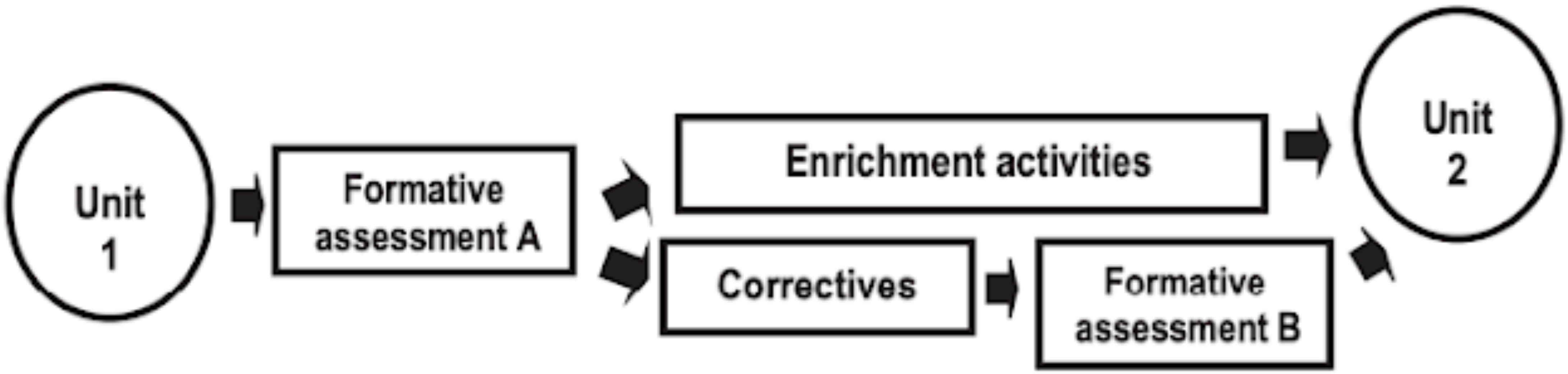
The Mathematics	▪ What's the big idea in this lesson? ▪ How does it connect to what I already know?
Cognitive Demand	▪ How long am I given to think, and to make sense of things? ▪ What happens when I get stuck? ▪ Am I invited to explain things, or just give answers?
Equitable Access to Mathematics	▪ Do I get to participate in meaningful mathematical learning? ▪ Can I hide or be ignored?
Agency, Ownership and Identity	▪ Do I get to explain, or present my ideas? Are they built on? ▪ Am I recognized as being capable and able to contribute in meaningful ways?
Formative Assessment	▪ Do classroom discussions include my thinking? ▪ Does instruction respond to my thinking and help me think more deeply?

(Schoenfeld, 2016)

FIGURE 1. *Achievement distribution for students under conventional, mastery learning, and tutorial instruction.*



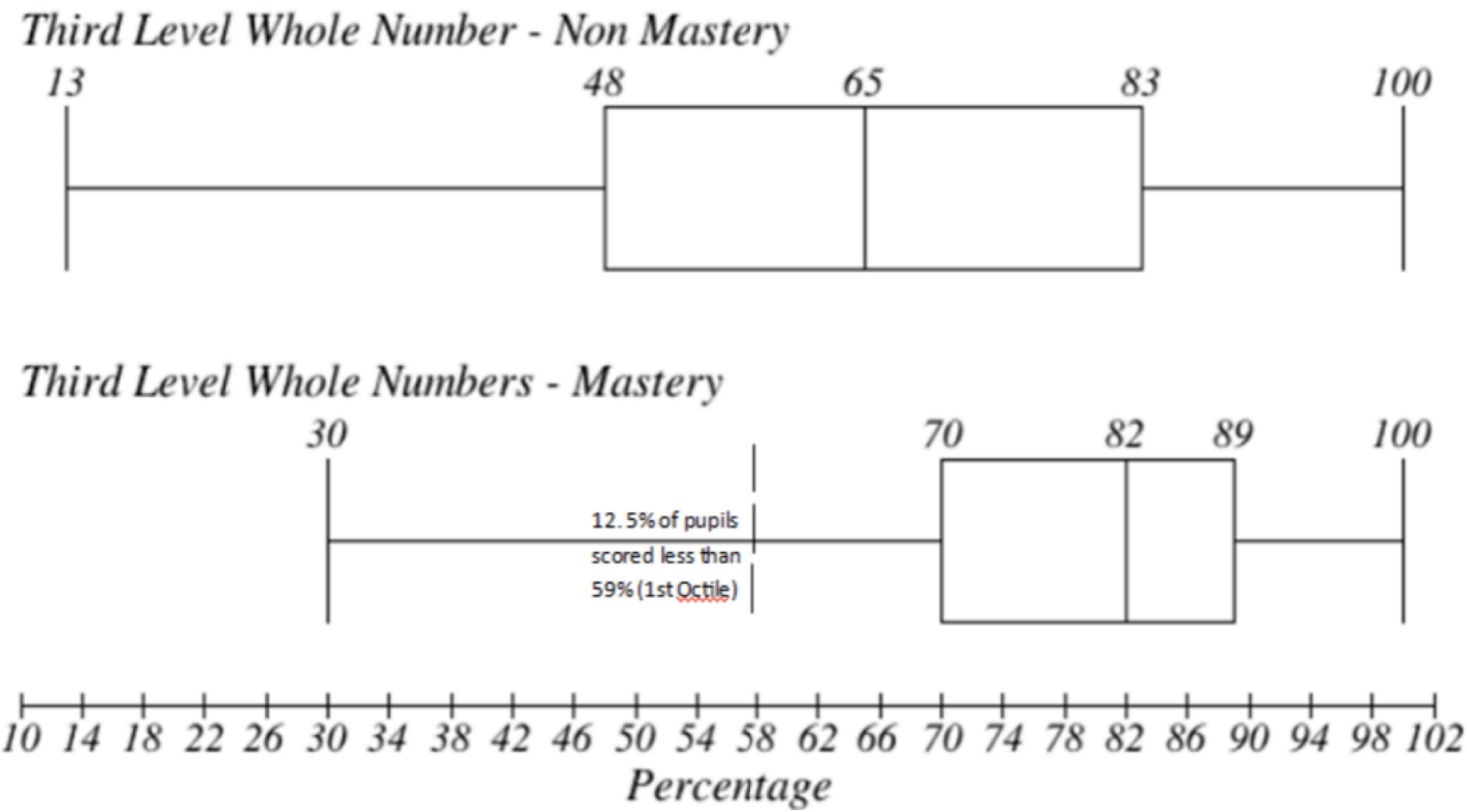
*Teacher-student ratio


Unit
1
Formative
assessment A
Enrichment activities
Correctives
Formative
assessment B
Unit
2

When compared to students in traditionally taught classes, students in well implemented mastery learning classes consistently:

- Reach higher levels of achievement
- Develop greater confidence in their ability to learn
- Develop greater confidence in themselves as learners

Anderson, 1994; Guskey & Pigott, 1988; Kulik, Kulik & Bangert-Drowns, 1990


Third Level Whole Number - Non Mastery
13
48
65
83
100
Third Level Whole Numbers - Mastery
30
70
82
89
100
12.5% of pupils
scored less than
59% (1st Octile)
10 14 18 22 26 30 34 38 42 46 50 54 58 62 66 70 74 78 82 86 90 94 98 102
Percentage

Arithmetic Essentials 1

PHASE 2 LEVEL 3

DIAGNOSTIC **ONE**

NAME: ....

MARKING GRID

Objective	Question	Score	Mastery Threshold	Practice Required? Yes/No
Add/Subtract	1	___ / 7	5	Y ☐ N ☐
Multiplication & Multiplicative Reasoning	2	___ / 5	4	Y ☐ N ☐
Division	3	___ / 5	4	Y ☐ N ☐
Multiply/Divide by multiples of 10,100,1000	4	___ / 4	3	Y ☐ N ☐
Long Multiplication	5	___ / 3	2	Y ☐ N ☐

OVERALL SCORE ___ / 24

MASTERY THRESHOLD 19 / 24

- Mathematics IS different

- 7 year gap upon arrival at secondary : significant numbers at 2 or 3 year gap.

- What about 'social justice'?

- Third Level / KS3 is the bridge from elementary arithmetic to abstract mathematical thinking of secondary (Tony Gardiner)

- Setting: Conveyor Belt versus Mastery

- Setting: Teacher expectations?

"A complete set of data containing all the explanatory variables was available for 3481 pupils. ***The analysis suggested that pupils made more progress if they were not in mixed ability schools,*** *and there was some evidence that set schools strengthened the relationship between Key Stage 2 and Key Stage 3 scores."*

Ability grouping in the secondary school: the effects on academic achievement and pupils' self-esteem

Judith Ireson, Susan Hallam, Peter Mortimore, Sarah Hack, Helen Clark and Ian Plewis

Institute of Education, University of London

Paper presented at the British Educational Research Association Annual Conference, University of Sussex at Brighton, September 2-5 1999

Dos and don'ts of setting

Do make setting as subject specific as possible

The negative effects of streaming – grouping students based on general 'ability' – on attainment and self-confidence are widely documented. The streaming approach also undermines the perceived benefits of attainment grouping (homogeneity of attainment in a group), given that students have different attainment for different subject areas. Instead, group students for maths according to maths attainment, for English by English attainment and so on.

Do group students by attainment only

Current attainment can be a reliable way of grouping students, based on what they know and can do. Other measures, such as 'effort' or 'attitude to work' are often influenced by negative stereotypes without teachers realising.

Do retest regularly and move students between groups

Students are motivated by the belief that when they work hard they will be rewarded by moving up a group. Regularly testing and moving students can act as an incentive and also destigmatise belonging to lower sets. Retesting and movement is also necessary to ensure that set groups reflect homogeneous attainment levels.

Do use a lottery system when assigning borderline students to sets

This mitigates the introduction of bias in assigning students from particular backgrounds to lower or higher sets.

Do make sure all students have access to a rich curriculum

Students in all sets will benefit from exciting subject knowledge and a wide range of activities. Being in a low set shouldn't mean you miss out on problem solving or creative opportunities.

Do apply high expectations to all sets

Keep expectations for learning opportunities, curriculum, behaviour and homework consistent and high across all sets.

Don't set by timetable convenience

Attainment grouping can be less fair when the timetable forces particular outcomes, for example preventing students from moving between groups. Aim to have a timetable that works to the benefit of students.

Don't extrapolate setting across subjects

If the timetable requires students to be in the same group for two or more subjects, you are no longer setting (rather, you are introducing elements of streaming). Students have different levels of attainment in different subjects, so a high attainer in English is not necessarily a high attainer in MFL. Hence linking subjects in such ways is likely to narrow opportunities for individual students as well as undermining the principle of subject-specific setting.

Don't assign subject expert teachers only to top sets

Lower sets can benefit greatly from subject experts, whose depth of understanding can help them explain subject material much more clearly.

Don't give less homework to low sets

Research has found that students in low sets tend to receive less homework. But being in a low set shouldn't mean fewer opportunities for learning development - and this includes homework.

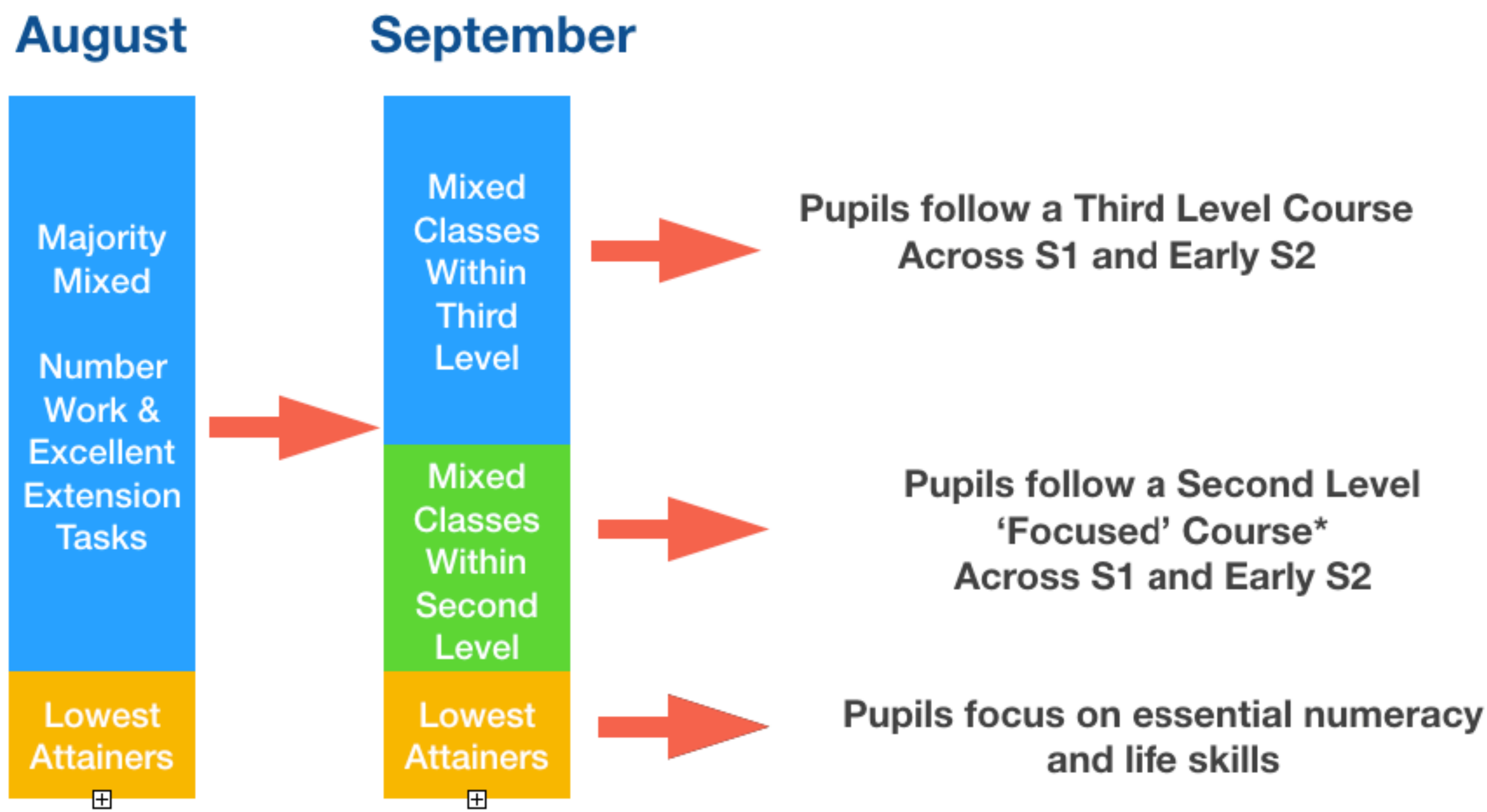
Don't provide low sets with a 'dumbed' down curriculum

If lower sets are taught a different curriculum from higher sets, it can be impossible for students to move groups, as well as impoverishing students' knowledge and skills. Make sure all students have access to a curriculum that gives them the best chances.

Don't leave students in sets without regular testing

Students can be motivated by knowing that they can move sets. It is more helpful for students to believe that they can improve their attainment through effort than that they have a fixed amount of 'ability' that means they need to stay in a low set.

To cite this resource: Francis, B., Taylor, B., Hodgen, J., Tereshchenko, A. & Archer, L. (2018).
Dos and don'ts of attainment grouping.
London: UCL Institute of Education.


August
September
Majority Mixed
Number Work & Excellent Extension Tasks
Lowest Attainers
Mixed Classes Within Third Level
Mixed Classes Within Second Level
Lowest Attainers
Pupils follow a Third Level Course Across S1 and Early S2
Pupils follow a Second Level 'Focused' Course* Across S1 and Early S2
Pupils focus on essential numeracy and life skills

A Scheme of Work

Can…	(Probably) Shouldn't….
Illustrate facts, procedures, concepts and behaviours to be learned	Be A list of Textbook Chapters
Be A Summary of Models of Didactics	Be A Replication of National Documentation
Be a Dynamic Document, Constantly Evolving	Be A Bureaucratic Tick Box Exercise
Contain a Bank of Quality Tasks and Questioning Prompts	Be 'Knocked Together' in a Couple of Weeks
Be A Collection of Assessment Tools	Be The Work of One Person
Be Informed by Evidence	Be A Timeline
Be Flexible, Not Tightly Scheduled	

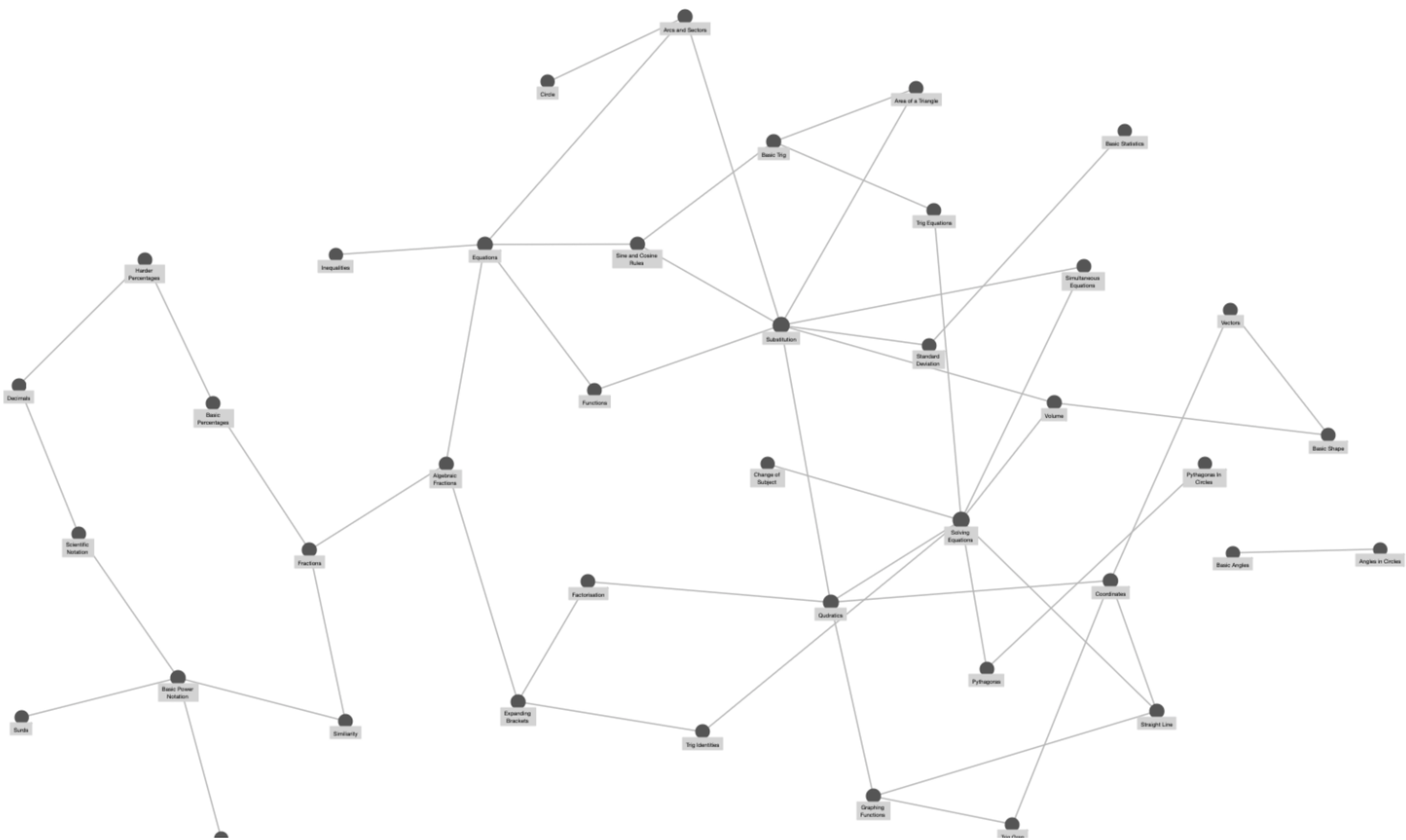
Previously : an improvement - but not enough.

CONTENT	RESOURCES	Comments/Teaching Advice	Assessment/Questioning	Rich Tasks
New Knowledge – Level 4				
• Be able to describe the gradient of a line	Teaching Slides Worksheet – Describe the gradient of a line	Note: Most pupils will see the gradient as the angle first which is true i.e. m = tanθ. However the angle is an inefficient way to quickly plot a straight line. STARTERS: Please include questions that allow pupils to remember the representations. e.g. Sketch a line where m>0 and c <0.	Show pupils an array of straight lines and ask them what makes them different. Ask pupils to group them into four categories. Use DESMOS to show pupils the difference between gradients. Link in slides Diagnostic – Describing the gradient	Teachers should use past paper questions through their chosen medium.
• Define the gradient of a line as vertical distance over horizontal distance.	Teaching Slides Worksheet 1 (State Gradient Counting Boxes) Worksheet 2 (State Gradient and Sketch Lines) Worksheet 3 (State Gradient by Stepping Method) Worksheet 4 (Mixed Questions)	National 4 requires pupils to find the gradient by comparing vertical distance with horizontal distance. It is advised that ALL pupils determine the gradient this way to begin as it can be made relevant to Higher pupils by using the step method (boxes & co-ordinates) before relying on formulae.	Diagnostic – Gradient (1 – v/h)	Teachers should use past paper questions through their chosen medium.
• • Be able to relate this to the gradient formula and calculate the gradient between any two points.	Teaching Slides Worksheet 1 – (Gradient using formula)	Teachers should encourage the notation of m_{AB} when working out the gradient between two named points i.e. A(1, 4) and B(-3, 7)	Diagnostic - Gradient (2 – y2-y1/x2-x1)	Teachers should use past paper questions through their chosen medium.
• Understand that the gradient alone cannot define a straight line and know that you need the placement on the cartesian diagram (y-intercept) in order to fully define a line.	Teaching Slides Worksheet – What is the y-intercept? (Sketch and relate the y-intercept) Worksheet – State gradient, y-int from equation (y = mx+c)		Diagnostic – Equation of a line (1)	Teachers should use past paper questions through their chosen medium.
• Be able to state the equation of a line from a diagram (where the y-intercept is depicted)	Teaching Slides Worksheet – State the equation/Sketch horizontal and vertical lines Worksheet – State the equation of a line from diagram with y-intercept shown			Teachers should use past paper questions through their chosen medium.
• Rearrange and state the gradient and y-intercept given any equation.	Teaching Slides Worksheet – Rearrange/State gradient & y-intercept		Diagnostic – Rearrange equation of line to find gradient & y-intercept	Teachers should use past paper questions through their chosen medium.

So that's where I start from on why I have trouble with tasks. The tasks may be beautiful, just like a meal may be beautiful, but that's not the problem. The problem is how you link whatever you do to the long term. Task design looks sensible because you know that even if you were to draw a map, you would have to set some exercises. You'd need to have the anchor task upfront. It feels like it's connected to the long-term challenge. But my observation is that it never is, it's separate. Malcolm Swan was absolutely excellent in embedding the tasks that he imagined/conceived/thought up or had presented to him into a kind of, I want to say a classroom setting or context. That would make wonderful sense if one were doing this chapter after chapter or square after square on a map in building up a progression – but that doesn't happen. As such, I don't see any alternatives to the textbook. Tasks encourage people to go and print out another bloody sheet from the web. I don't know any settings in which that works at scale. I can imagine it working in the odd classroom with the odd teacher but…I'm stuck."

Learning Trajectories

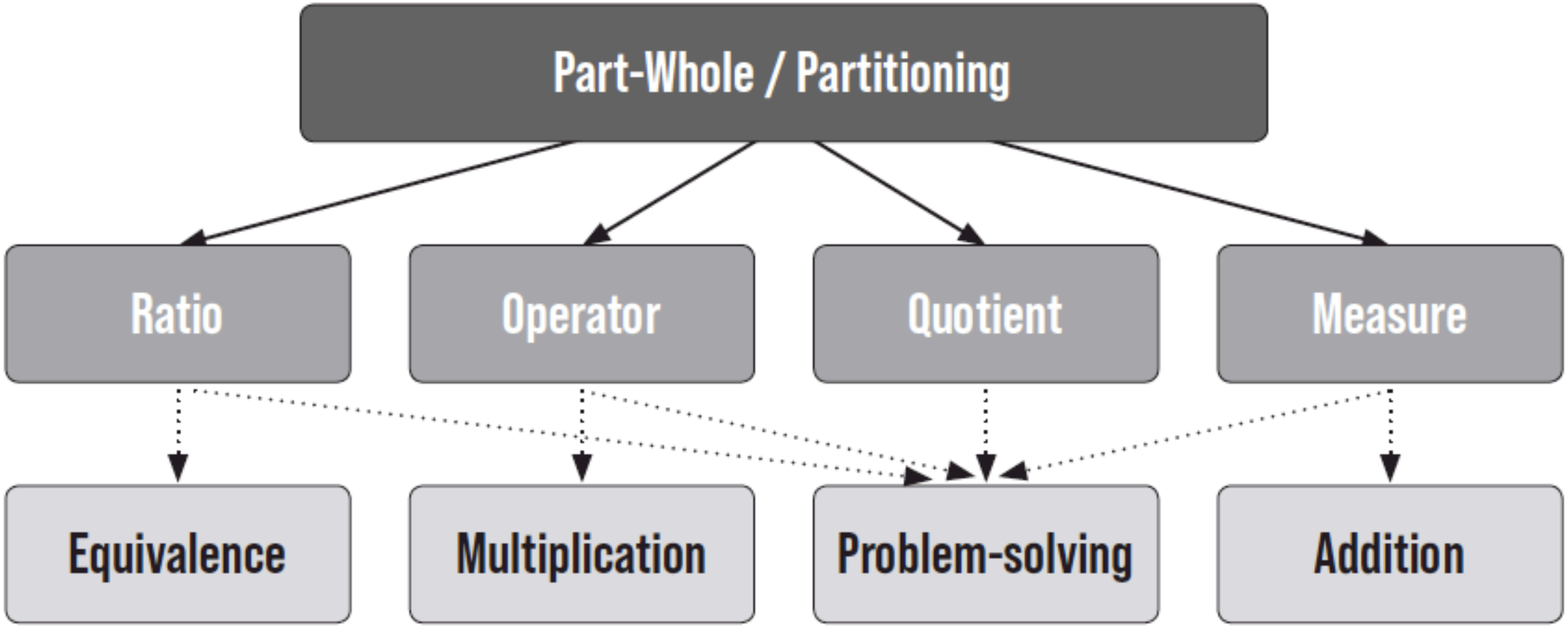
starttingpointsmaths.com



Martin Simon (1995) describes learning trajectories as having three components:

1. A set of mathematical goals.
2. A clearly marked developmental path.
3. A coherent set of instructional tasks or activities.

Kyong Mi Choi discusses how textbook tasks in South Korea are developed by teams of mathematicians, mathematics educators and mathematics teachers to ensure mathematical correctness, and to consider how learners understand mathematics and possible misconceptions. These tasks have five core elements – intuitive exploration, explanation, examples, practice, and extension – and there is a richness to the pedagogy. To promote intuitive exploration of ideas, tasks borrow concepts from everyday life, ask pupils to fill in a missing piece of reasoning, or use mathematical knowledge from earlier grades (Ferreras, Kessel and Kim, 2015).



(Silver, Behr, Post and Lesh, 1983)

- Is an over-emphasis on procedural fluency is obstructive ?

- **First Level:** Calculates simple fractions of a quantity and uses this knowledge to solve problems in everyday contexts, for example, find $\frac{3}{5}$ of 60.
- **Second Level:** Uses knowledge of fractions, decimal fractions and percentages to carry out calculations with or without a calculator.
- **Third Level:** Applies addition, subtraction and multiplication skills to solve problems involving fractions and mixed numbers.

Relationship between fractions, multiplication and division: There is a direct link between finding a fraction of an object or a quantity and multiplication and division.

(Scottish Government, 2018)

1. Repeated addition: area model.
2. Multiplication as operator: measurement and set model.
3. Taking a part of a part of a whole: area model.

The comparison of approaches is important!

Developmental Path (Mathematical Goals)	Instructional Tasks
1. To understand (whole number x proper fraction) and (whole number x mixed number) calculate in multiple ways.	There are five pizzas of which $\frac{3}{8}$ of each pizza remain. How much pizza is there in total?
2. To understand (proper fraction x whole number) with manipulative; formulate algorithm and use it proficiently. To understand that if a multiplier is smaller than 1, then the product is smaller than the multiplicand.	12 m of wire was bought to make a wire sculptured animal with clay. If ¾ of the wire is used, how many m of the wire is used?
3. To understand (proper fraction x mixed number) with manipulative; calculate with two methods and compare two methods.	$12\frac{3}{4}$ m of wire was bought to make a wire sculptured animal with clay. If $\frac{3}{2}$ of the wire is used, how many m of the wire is used?

(Ferreras, Kessel and Kim, 2015)

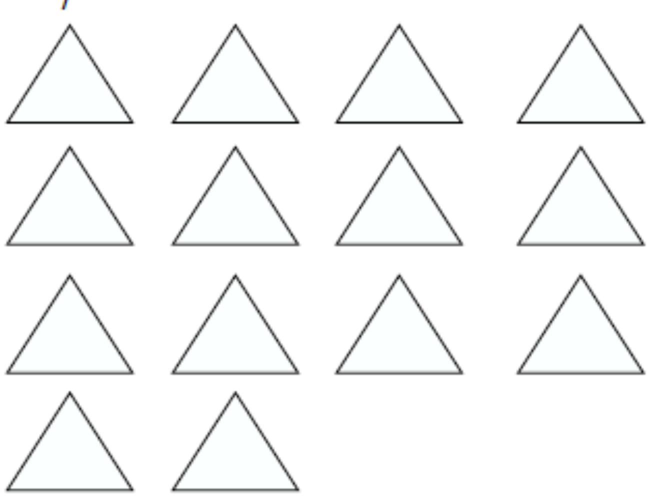
The Story of $\frac{4}{7}$

Shade $\frac{4}{7}$

Shade $\frac{4}{7}$

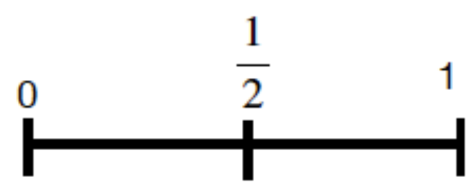
Shade $\frac{4}{7}$

Shade $\frac{4}{7}$



Write $\frac{4}{7}$ in words.

Locate it:



Find $\frac{4}{7}$ *of* 63

Find $\frac{4}{7} \times 21$

What division does $\frac{4}{7}$ represent?

What is the quotient?

I have 4 apples and 7 bananas.
Fill in the missing fractions.

The number of apples = of the fruit

The number of bananas = of the fruit

The number of apples = of the bananas

The number of bananas = of the apples

Write 3 fractions which are equivalent to $\frac{4}{7}$

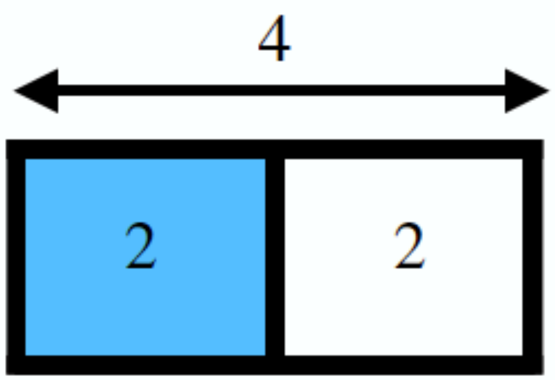
Write down two fractions which add to give $\frac{4}{7}$

Division	Written Calculation	Mixed Numbers	Improper
$13 \div 5$	$5 \overline{)\,13}$ 2 r 3	$2\frac{3}{5}$	$\frac{13}{5}$
$11 \div 4$			
$12 \div 7$			

Question	Visual	Working	Answer
Find $\frac{1}{3}$ of 12	4 | 4 | 4	$12 \div 3$	£4
Find $\frac{2}{3}$ of 12	4 | 4 | 4	$12 \div 3 = 4$ $4 \times 2 = 8$	
Find $\frac{3}{3}$ of 12	4 | 4 | 4		

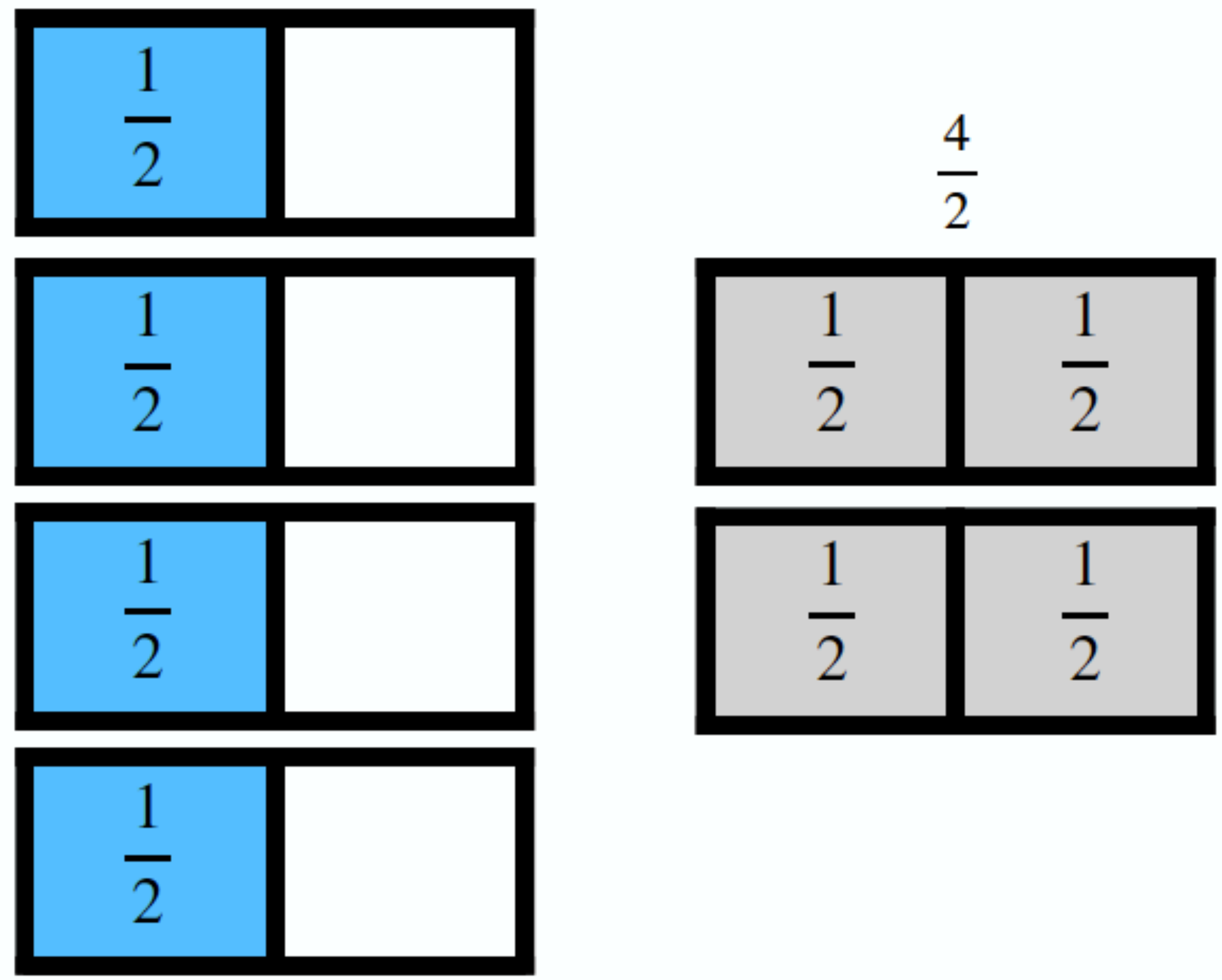
Example

$\frac{1}{2}$ *of* 4



$\frac{1}{2}$ *of* $4 = 2$

$\frac{1}{2} \times 4$



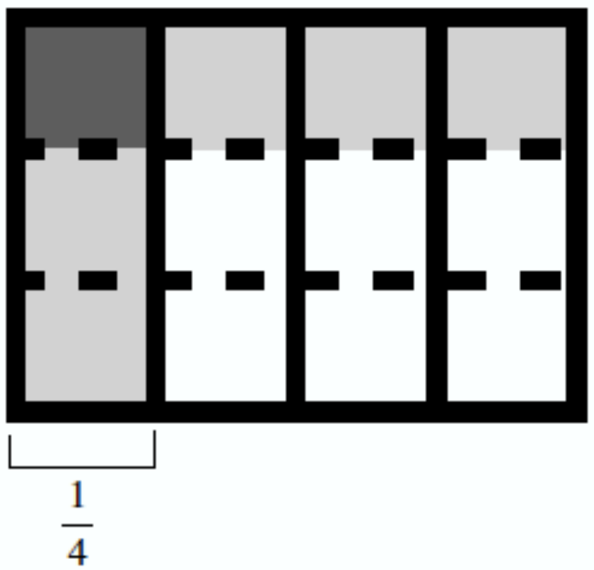
$\frac{1}{2} \times 4 = \frac{4}{2} = 2$

1 Calculate the following, drawing bar models for working:

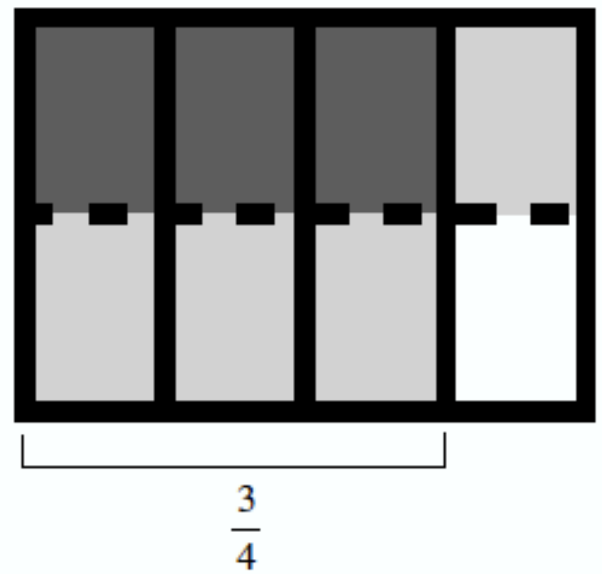
(a) $\frac{1}{2} \times 6$ (b) $8 \times \frac{1}{4}$ (c) $\frac{1}{3}$ *of* 6 (d) $\frac{1}{5}$ *of* 10

1 Use the following diagrams to help you multiply the fractions:

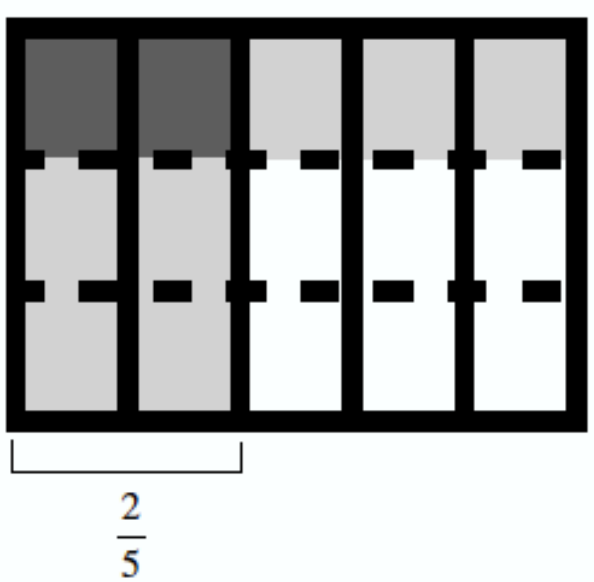
(a) $\frac{1}{3} \times \frac{1}{4}$



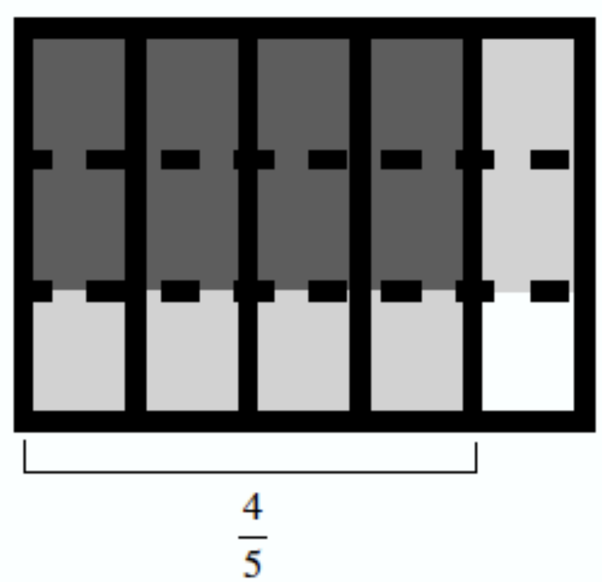
(b) $\frac{1}{2} \times \frac{3}{4}$



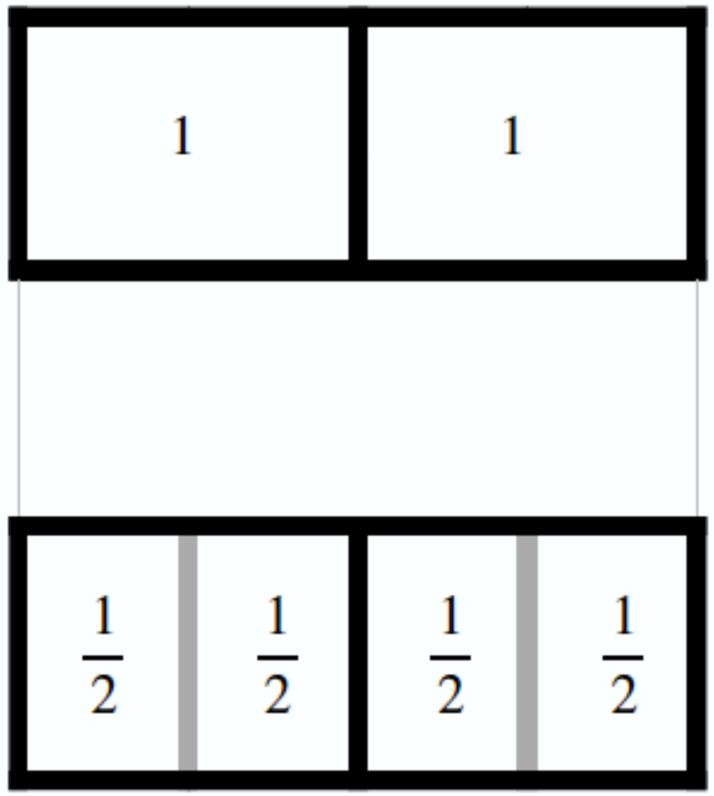
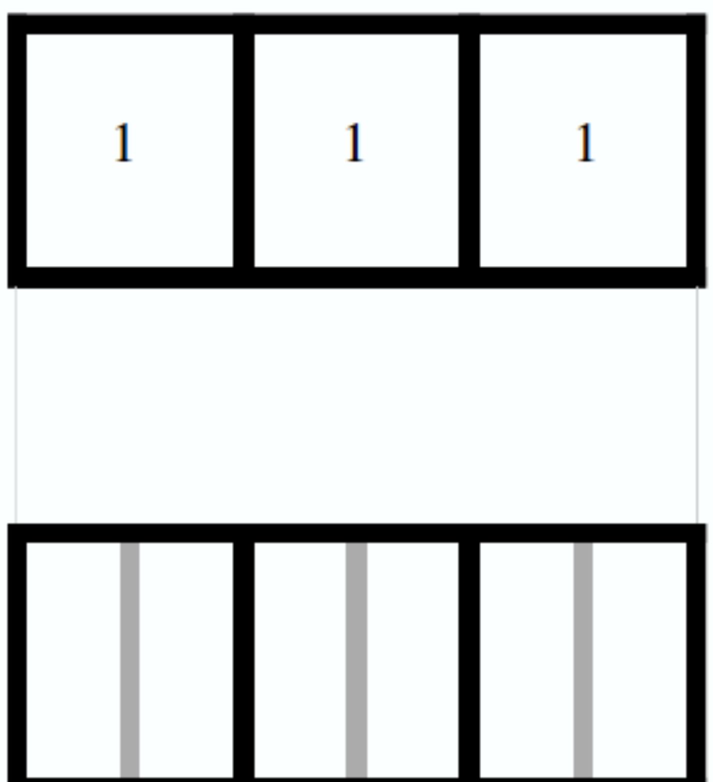
(c) $\frac{1}{3} \times \frac{2}{5}$



(d) $\frac{2}{3} \times \frac{4}{5}$



2. Complete the pattern and answer the questions below:

(a)	$2 \div \frac{1}{2}$		How many halves are in 2?	$2 \div \frac{1}{2} =$
(b)	$3 \div \frac{1}{2}$		How many halves are in 3?	

$\frac{1}{5} \div 5$	$\frac{1}{4} \div 5$	$\frac{1}{3} \div 5$	$\frac{1}{2} \div 5$	$1 \div 5$	$2 \div 5$	$3 \div 5$	$4 \div 5$	$5 \div 5$
$\frac{1}{5} \div 4$	$\frac{1}{4} \div 4$	$\frac{1}{3} \div 4$	$\frac{1}{2} \div 4$	$1 \div 4$	$2 \div 4$	$3 \div 4$	$4 \div 4$	$5 \div 4$
$\frac{1}{5} \div 3$	$\frac{1}{4} \div 3$	$\frac{1}{3} \div 3$	$\frac{1}{2} \div 3$	$1 \div 3$	$2 \div 3$	$3 \div 3$	$4 \div 3$	$5 \div 3$
$\frac{1}{5} \div 2$	$\frac{1}{4} \div 2$	$\frac{1}{3} \div 2$	$\frac{1}{2} \div 2$	$1 \div 2$	$2 \div 2$	$3 \div 2$	$4 \div 2$	$5 \div 2$
$\frac{1}{5} \div 1$	$\frac{1}{4} \div 1$	$\frac{1}{3} \div 1$	$\frac{1}{2} \div 1$	$1 \div 1$	$2 \div 1$	$3 \div 1$	$4 \div 1$	$5 \div 1$
$\frac{1}{5} \div \frac{1}{2}$	$\frac{1}{4} \div \frac{1}{2}$	$\frac{1}{3} \div \frac{1}{2}$	$\frac{1}{2} \div \frac{1}{2}$	$1 \div \frac{1}{2}$	$2 \div \frac{1}{2}$	$3 \div \frac{1}{2}$	$4 \div \frac{1}{2}$	$5 \div \frac{1}{2}$
$\frac{1}{5} \div \frac{1}{3}$	$\frac{1}{4} \div \frac{1}{3}$	$\frac{1}{3} \div \frac{1}{3}$	$\frac{1}{2} \div \frac{1}{3}$	$1 \div \frac{1}{3}$	$2 \div \frac{1}{3}$	$3 \div \frac{1}{3}$	$4 \div \frac{1}{3}$	$5 \div \frac{1}{3}$
$\frac{1}{5} \div \frac{1}{4}$	$\frac{1}{4} \div \frac{1}{4}$	$\frac{1}{3} \div \frac{1}{4}$	$\frac{1}{2} \div \frac{1}{4}$	$1 \div \frac{1}{4}$	$2 \div \frac{1}{4}$	$3 \div \frac{1}{4}$	$4 \div \frac{1}{4}$	$5 \div \frac{1}{4}$
$\frac{1}{5} \div \frac{1}{5}$	$\frac{1}{4} \div \frac{1}{5}$	$\frac{1}{3} \div \frac{1}{5}$	$\frac{1}{2} \div \frac{1}{5}$	$1 \div \frac{1}{5}$	$2 \div \frac{1}{5}$	$3 \div \frac{1}{5}$	$4 \div \frac{1}{5}$	$5 \div \frac{1}{5}$

Levels of cognitive demand	Characteristics of tasks
Level 0 – Very low Memorisation tasks	▪ Reproduction of facts, rules, formulae ▪ No explanations required
Level 1 – Low Procedural tasks without connections	▪ Algorithmic in nature ▪ Focused on producing correct answers ▪ Typical textbook word problems ▪ No explanations required
Level 2 – High Procedural tasks with connections	▪ Algorithmic in nature ▪ Has a meaningful / 'real world' context ▪ Explanations required
Level 3 – Very High Problem solving / doing mathematics	▪ Non-algorithmic in nature, requires understanding and application of mathematical concepts ▪ Has a 'real world' context / a mathematical structure ▪ Explanations required

(Kaur, Wong and Chew, 2018)

Food for Thought	
Junk food	*Junk mathematics*
There is a lot of it about.	See most school textbooks.
All the preparation is done for you by someone else.	This is done by the author or teacher – all the nasties are removed.
The instructions for use are simple and laid out in steps.	See most textbook questions.
It is superficially attractive but turns out to lack flavour.	It looks well-structured and appears logical, but is dull and lacks substance.
It does you little good; it tends to pass through quickly.	Pupils are unable to retain or apply it in new contexts.
All the real nutrient is removed and substitutes have to be added.	It offers no real life situations but invents and contrives them.
DANGER: HEALTH WARNING **Junk mathematics can seriously damage your pupils.**	

Phase 1 - mixed classes
Area
Length
Order of Operations
Fractions

Phase 2
Arithmetic Essentials (1)
Integers (1)
Factors & Primes
Angles (1)
Fractions (1)
Assessment and Revision Materials

Phase 3
Arithmetic Essentials (2)
Integers (2)
Fractions (2)
Expressions (1)
Data Analysis

Phase 4
Angles (2)
Further Expressions & Substitution.
Percentages
Solving Equations (1)

Phase 5
Link
Ratio
Solving Equations (2)
Area (2) & Volume
Coordinates & Sequences

Pre-Requisite Topic	% Questions appearing in
Equations	28.5%
Substitution	28.5%
Expressions	25%
Integers	21%
Fractions	17%
Co-ordinates	11%
Area of basic shapes	11%

- **Forward Facing**
- **Connectionist**
- **Atomisation?**

- Not just random sheets
- A planned journey
- Coherence of models and metaphors
- A rich variety of taks: procedural, conceptual and problem solving
- To convey the curriculum to teachers
- To guide the pedagogical approaches

Negative Numbers

PHASE 2 LEVEL 3

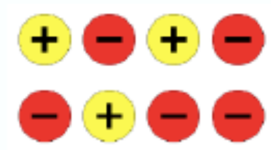
BOOK **ONE**

HOLYROOD SECONDARY SCHOOL

MATHEMATICS DEPARTMENT

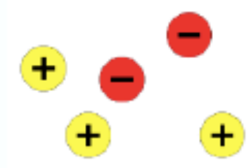
1. For each diagram state if the value is zero. True or False.

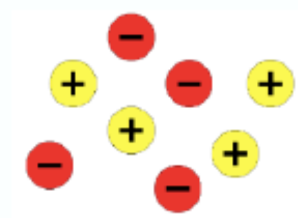
(a) 

(b) 

(c) 

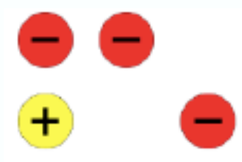
(d) 

(e) 

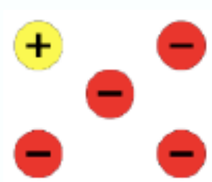
(f) 

2. Add extra counters to each of the diagrams below so that they equal zero.

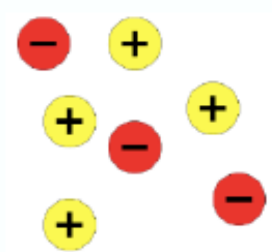
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

3. Complete each statement below:

(a) $5 + {}^-5 = \square$

(b) $3 + \square = 0$

(c) ${}^-7 + \square = 0$

(d) $\square + {}^-4 = 0$

(e) $\square + 2 = 0$

(f) $a + \square = 0$

4. Write down the additive inverse of each number

(a) 5 (b) $^-4$

(c) $^-40$ (d) 12

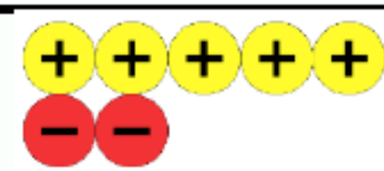
(e) $\frac{1}{2}$ (f) $^-0.56$

(g) a (h) ^-a

Adding Integers

Representing Addition

1 Complete the table below.

Question	Re-written	Using Tiles	In words	Answer
5 + (-2)	$^{+}5 + {}^{-}2$		positive 5 plus negative 2	3
4+ (-3)				
(-5) + 3				
-2 + 1				
3+ (-4)				
2+ -5				
(-3)+ 4				
-2+ 3				
(-2) + (-3)				
-3 + -4				

1. Answer the questions below and reflect on your answers.

(a)		(b)		(c)		(d)	
	8 + 3 =		-3 + 5 =		-6 + 1 =		-8 + 2 =
	8 + 2 =		-3 + 4 =		-6 + 2 =		-8 + 1 =
	8 + 1 =		-3 + 3 =		-6 + 3 =		-8 + 0 =
	8 + 0 =		-3 + 2 =		-6 + 4 =		-8 + -1 =
	8 + -1 =		-3 + 1 =		-6 + 5 =		-8 + -2 =
	8 + -2 =		-3 + 0 =		-6 + 6 =		-8 + -3 =
	8 + -3 =		-3 + -1 =		-6 + 7 =		-8 + -4 =

2. Do the following mentally, then check your answer by drawing a diagram.

(a) -4 + 6
(b) -5 + 8
(c) -1 + 3
(d) -4 + 7
(e) 2 + (-3)
(f) -1 + (-4)
(g) -2 + (-3)
(h) -6 + 6
(i) -7 + 4
(j) -6 + 2
(k) -7 + 2
(l) 5 + (-5)

3. Calculate the following.

(a)	5 + (-5)	(a)	6 + (-4)
(b)	(-7) + 7	(b)	6 + (-8)
(c)	5 + (-4)	(c)	(-4) + (-4)
(d)	6 + (-7)	(d)	(-2) + (-2)
(e)	(-7) + 8	(e)	(-4) + (-3)
(f)	(-3) + 2	(f)	(-2) + (-1)
(g)	(-5) + 7	(g)	(-6) + (-4)
(h)	(-5) + 3		

4. Complete each of the following tables.

+	-4	-2	0	2	4
-3					
-1					
1					
3					

+		1		3	
-5	-5				-1
		-3			
				0	
	-2		0		

Adding Integers

Generalising

1 Create an example of an addition which belongs in each box.
Some boxes are not possible to fill in - decide which ones.

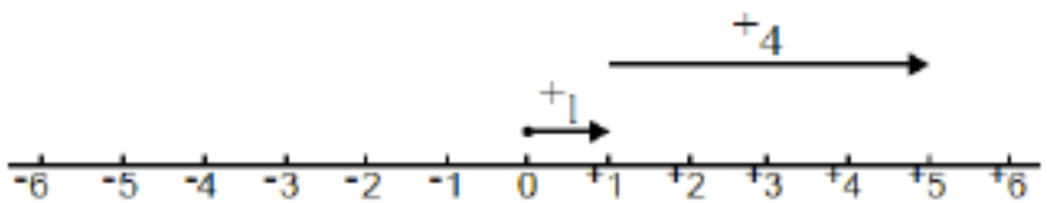
	Sum is **positive**	Sum is **zero**	Sum is **negative**
positive + positive			
negative + positive			
positive + negative			
negative + negative			

Adding Integers

Number Line

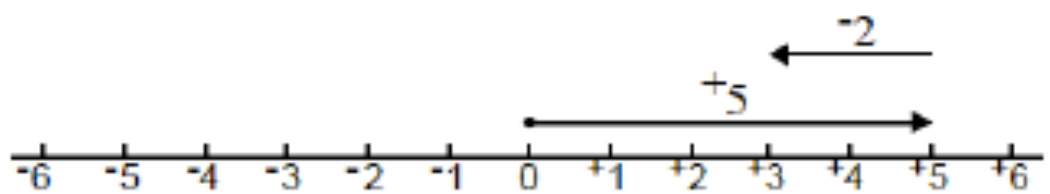
Add these shifts (*the first one is done for you*).

1.



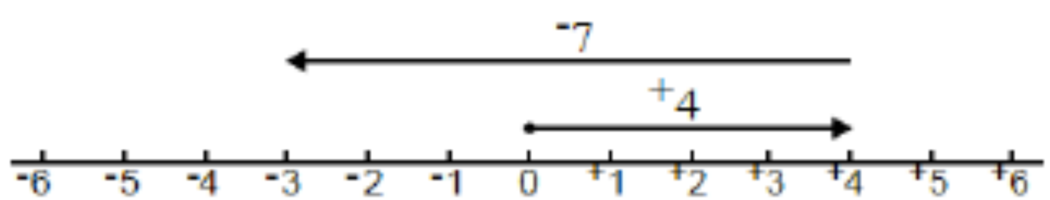
$\boxed{^{+}1} + \boxed{^{+}4} = \boxed{^{+}5}$

2.



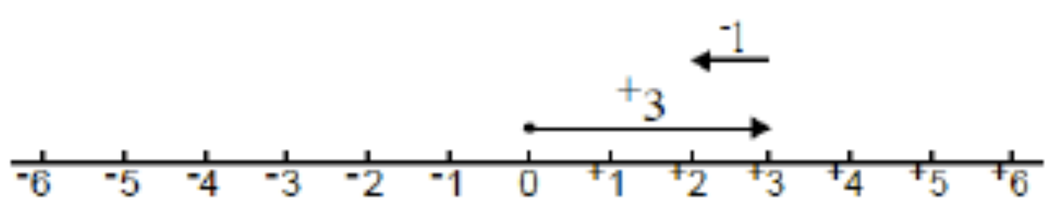
$\boxed{^{+}5} + \boxed{^{-}2} = \boxed{}$

3.



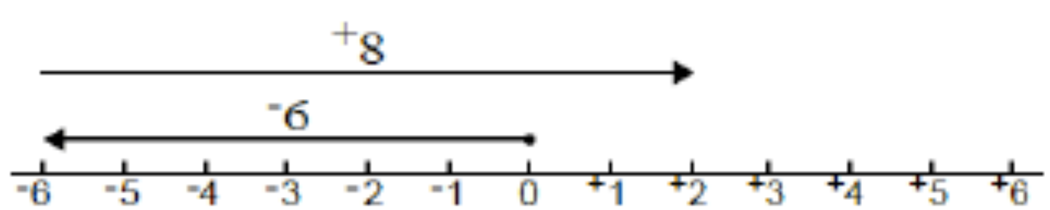
$\boxed{} + \boxed{} = \boxed{}$

4.



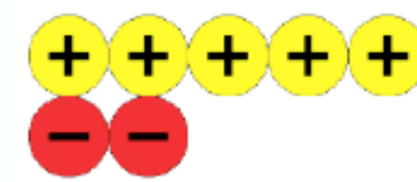
$\boxed{} + \boxed{} = \boxed{}$

5.

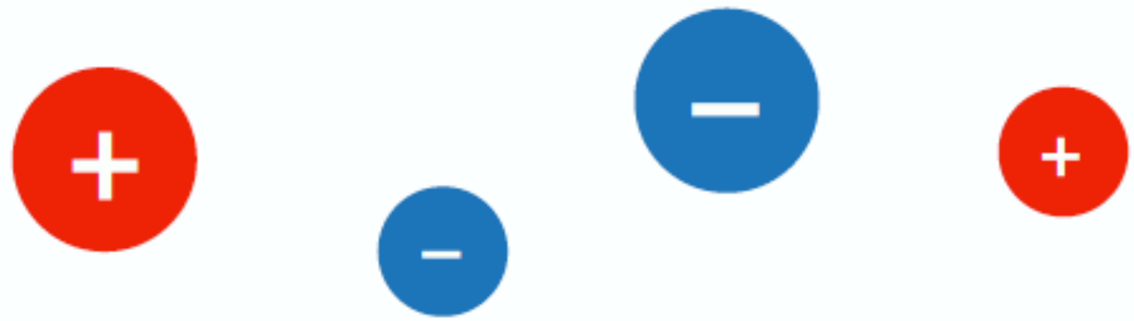


$\boxed{} + \boxed{} = \boxed{}$

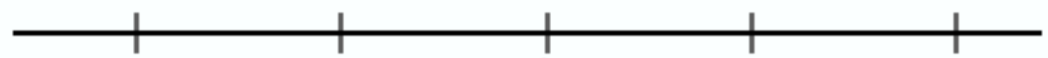
1 Complete the table below.

Question	In words	Re-written	Using Tiles	Answer
5 – 2	positive 5 minus positive 2	$5 + {}^{-}2$		3
4 – 3				
3 – (-2)	positive 3 minus negative 2	3 + 2		
2 – (-4)				
(-3) – 5		${}^{-}3 + {}^{-}5$		
(-2) – 3				
(-4) – 2				

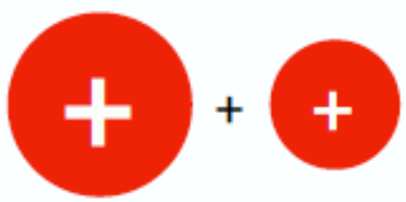
Some numbers are represented by the counters below.

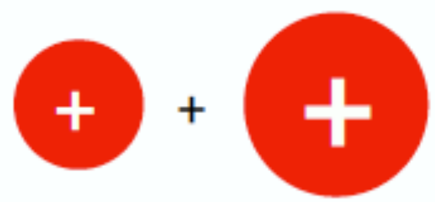


Complete the empty number line to show the possible positions of the numbers.



State if each of the following calculations has an answer which is positive, negative or zero.

(a) 

(b) 

(c) 

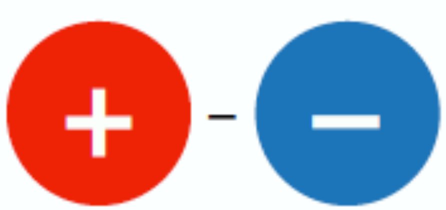
(d) 

(e) 

(f) 

(g) 

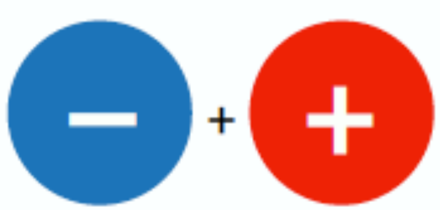
(h) 

(i) 

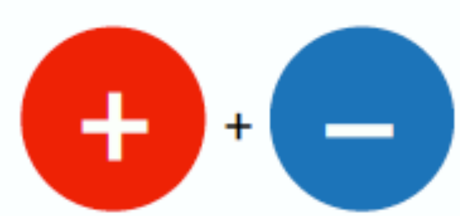
(j) 

(k) 

(l) 

(m) 

(n) 

(o) 

(p) 

1. For the given selection of integers:

-1	2	-3	4	-5

Use addition and subtraction of a combination of these numbers to make our target: 9.

Use each given integer no more than once.

A couple of examples have been done for you. Can you think of two more ways?

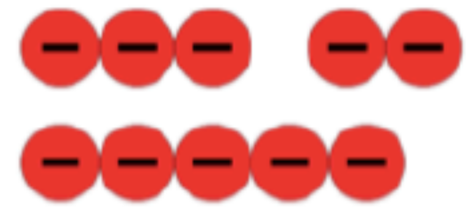
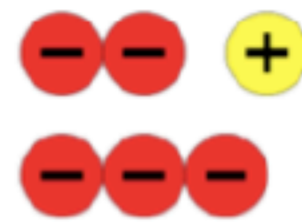
$$5 - (-4) = 9$$

$$2 + 4 - (-3) = 9$$

2. Using the same selection of integers, create combinations of addition and subtraction of the numbers for each given target. Use each integer no more than once.

Target	Calculation
1	
-2	
3	
-4	
5	
0	
12	
-15	
15	

Combining Multiple Terms: Scaffolded Practice

Question	Re-written	Using Counters	Answer
- 3 - 2 - 5	$^{-}3 + {}^{-}2 + {}^{-}5$		
- 2 - (-1) - 3	$^{-}2 + 1 + {}^{-}3$		
1 - 2 - 3			
- 1 + 2 - 3			
-5 + 3 - 2			
5 - 10 + 3			

Multiplication of Negatives — Practice

1. Calculate the following:

(a) $(-7)\times 2$ (b) $(-4)\times 8$ (c) $(-2)\times(-5)$

(d) $(-6)\times(-3)$ (e) $(-3)\times 7$ (f) $(-10)\times(-4)$

(g) 8×4 (h) $3\times(-6)$ (i) $(-7)\times(-2)$

(j) $(-4)\times(-5)$ (k) $(-7)\times 0$ (l) $8\times(-5)$

2. Calculate the following:

(a) $3\times(-8)\times(-4)$ (b) $(-4)\times(-8)\times(-2)$

(c) $(-2)\times(-2)\times 2$ (d) $4\times(-7)\times 2$

(e) $(-2)\times 8\times(-4)$ (f) $(-6)\times(-2)\times(-1)$

3. Write down the next 3 terms in each sequence:

For each sequence write down the rule that is used to calculate the next term.

(a) $1,\ -2,\ 4,\ -8,\ 16,\ \dots$

(b) $-1,\ 2,\ -4,\ 8,\ -16,\ \dots$

(c) $1,\ -10,\ 100,\ -1000,\ \dots$

(d) $1,\ -3,\ 9,\ -27,\ \dots$

(e) $-1,\ 5,\ -25,\ 125,\ \dots$

4. Complete each of the multiplication grids

(a)

×	1	0	−1	−2	−3
−4					
−2					
0					
1					

(b)

×		−1	
2			
	−2	2	
−3			−9

(c)

×	−2		
	10		
−2		6	
3			−12

Powers and roots of negatives

Revision

1. Evaluate each of the following:

(a) 2^2 (b) 2^3 (c) 2^4 (d) 2^5 (e) 2^6

(f) $(-2)^2$ (g) $(-2)^3$ (h) $(-2)^4$ (i) $(-2)^5$ (j) $(-2)^6$

(k) $(-3)^2$ (l) $(-3)^3$ (m) $(-3)^4$ (n) $(-3)^5$ (o) $(-3)^6$

2. State whether each of the following is positive or negative. Do not calculate the answers.

(a) $(-17)^2$ (b) $(-45)^3$ (c) $(-72)^4$ (d) $(-9)^5$ (e) 10^5

3. Evaluate each of the following:

(a) $\sqrt{25}$ (b) $\sqrt[3]{8}$ (c) $\sqrt[4]{81}$ (d) $\sqrt[5]{32}$ (e) $\sqrt[6]{1{,}000{,}000}$

(f) $\sqrt[3]{-8}$ (g) $\sqrt[3]{-1000}$ (c) $\sqrt[5]{-32}$ (d) $\sqrt[5]{-1}$ (e) $\sqrt[7]{-128}$

4. Peter claims that: It is possible to square root a negative number

Explain, using an example, why he is incorrect.

Division (and multiplication) of negatives

1. Calculate the following:

(a) $(-10) \div (-2)$ (b) $(-15) \div 5$ (c) $18 \div (-3)$

(d) $14 \div (-7)$ (e) $(-21) \div (-3)$ (f) $(-45) \div 9$

(g) $50 \div (-5)$ (h) $50 \div (-5)$ (i) $\frac{(-100)}{(-4)}$

(j) $\frac{80}{-2}$ (k) $\frac{(-26)}{13}$ (l) $\frac{(-70)}{(-7)}$

2. Complete the following:

(a) $\ldots\ldots \times 5 = -20$ (b) $(-80) \div \ldots\ldots = 4$

(c) $16 \times \ldots\ldots = -32$ (d) $(-4) \times \ldots\ldots = 32$

(e) $\ldots\ldots \times (-3) = 12$ (f) $40 \div \ldots\ldots = -8$

(g) $-8 \times \ldots\ldots = 48$ (h) $-32 \div \ldots\ldots = 4$

(i) $15 \times \ldots\ldots = -60$ (j) $100 \div \ldots\ldots = -25$

3. Calculate the following:

(a) $\frac{(-3) \times (-4)}{(-2)}$ (b) $\frac{5 \times (-6)}{(-2)}$ (c) $\frac{(-7) \times (-5) \times (-2)}{5}$

(d) $\frac{8 \times (-9) \times 6}{(-2) \times (-3)}$ (e) $\frac{(-6) \times (-4)}{2}$ (f) $\frac{(-4) \times (-7) \times 3}{(-12)}$

4. Calculate the following:

(a) $(-6 + 10) \div (-2)$ (b) $(12 - 24) \div (-2)$

(c) $(6 + (-8)) \times (4 - 7)$ (d) $((-2) + 8) \times ((-4) + 2)$

(e) $((-4) \times 2) + (6 \times (-9))$ (f) $(8 \times (-2)) - ((-4) \times 8)$

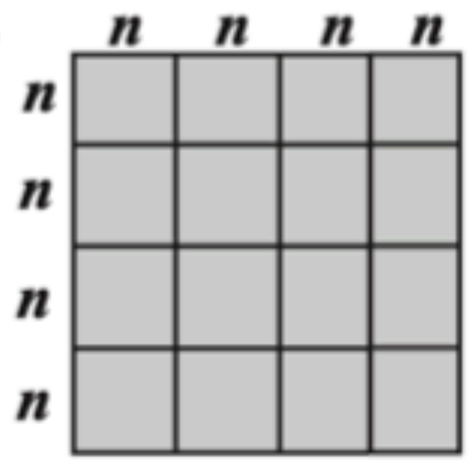
5. Calculate the following:

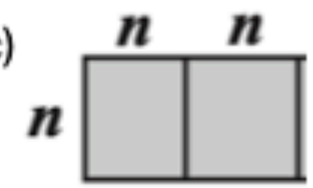
(a) $(-6) \times (-3) + (-4)$ (b) $(-5) \times 4 - (-3)$

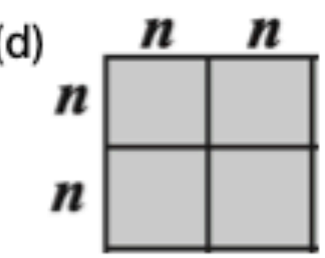
(c) $(-8) \times (-7) - 8 \times 7$ (d) $(-11) \times 4 + (-8) \times (-3)$

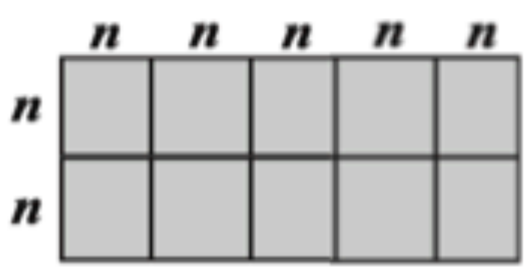
1. Write an expression for each of the diagrams below.

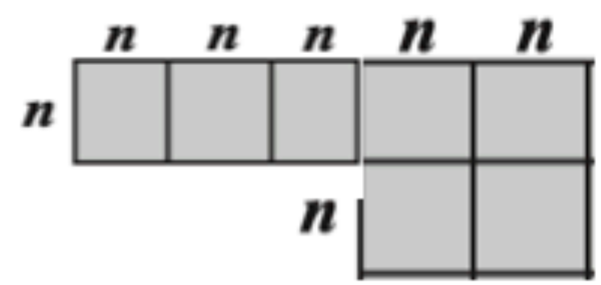
(a)


(b)


(c)


(d)


(e)


(f)


2. Draw a diagram for each of the following expressions:

(a) $5a^2$ (b) $(5a)^2$ (c) $6b^2$ (d) $(6b)^2$ (e) $2c^2 + (3c)^2$

Examples

If $a = 3$ and $b = 4$, calculate the values of: (a) $4a^2$ (b) $2a^2b$ (c) $(5ab)^2$

(a) $4a^2$
$= 4 \times 3^2$
$= 4 \times 9$
$= 36$

(b) $2a^2b$
$= 2 \times 3^2 \times 4$
$= 2 \times 9 \times 4$
$= 72$

(c) $(5ab)^2$
$= (5 \times 3 \times 4)^2$
$= (60)^2$
$= 3600$

3. If $x = 3$, $y = 2$ and $z = 6$, calculate the values of:

(a) $2x^2$ (b) $(2x)^2$ (c) $(3y)^2$ (d) $3y^2$ (e) $\frac{1}{2}z^2$

(f) $(\frac{1}{2}z)^2$ (g) $(xy)^2$ (h) xy^2 (i) x^2y (j) x^2y^2

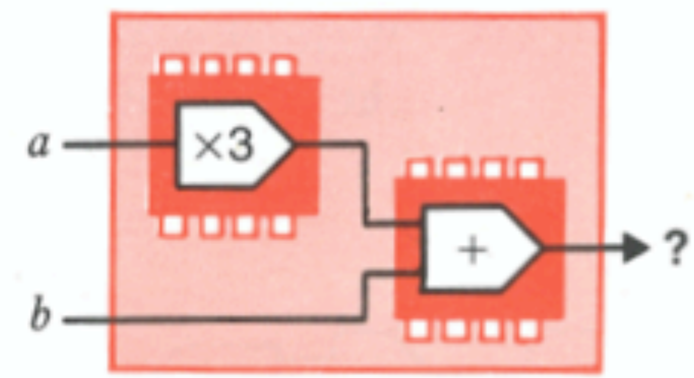
Example

(a) What is the output from this circuit if a = 2 and b = 5?

$$3 \times 2 + 5 \quad = 11$$

(b) Write the output of this circuit as an algebraic expression.

$$3a + b$$

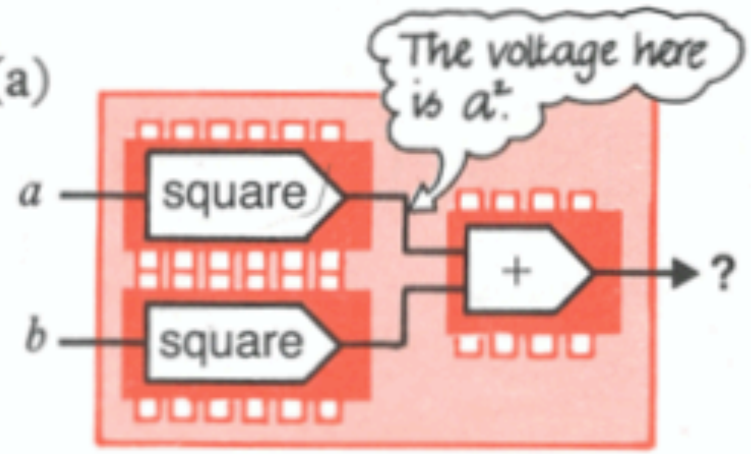


(c) Check this algebraic expression by substituting a = 2 and b = 5

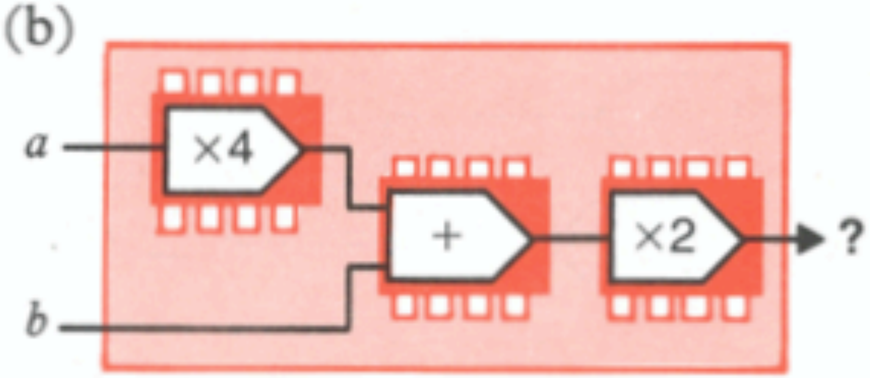
$$3a + b$$
$$= 3 \times 2 + 5$$
$$= 11$$

1. Write the output of each of circuit as an algebraic expression. Check using numbers like in the example.

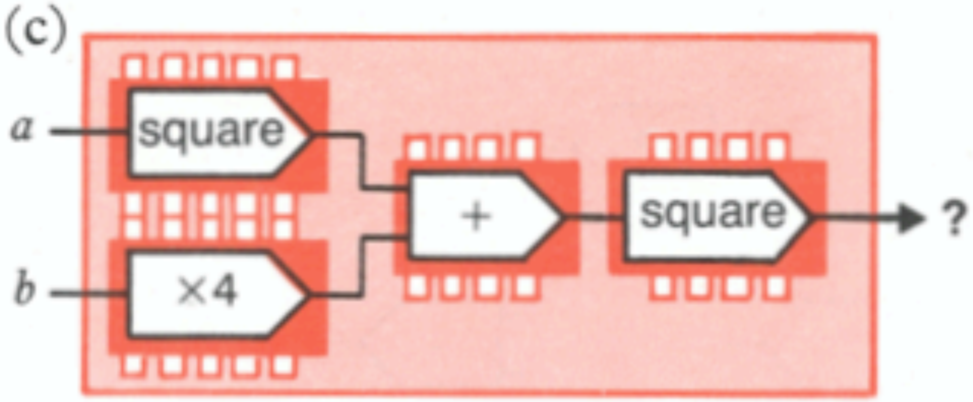
(a)



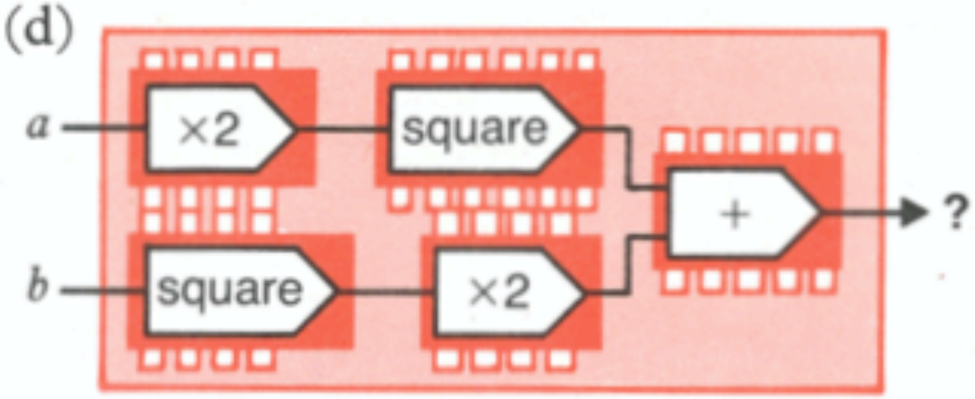
(b)

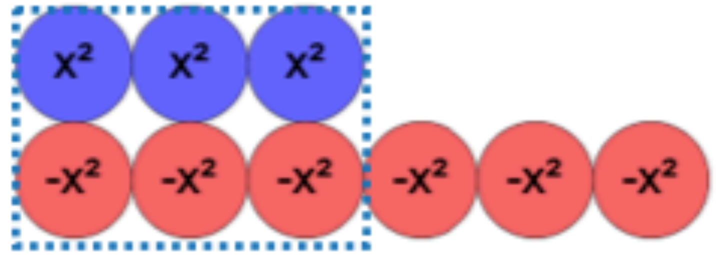
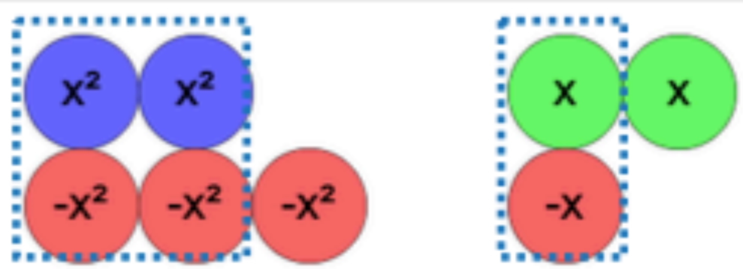


(c)



(d)



Expression	Diagram	Simplified
$3x^2 - 5x^2 - x^2$		$-3x^2$
$x^2 + x^2$		
$3x^2 - 2x^2 + x^2$		
$-3x^2 + 2x^2 - x^2$		
		

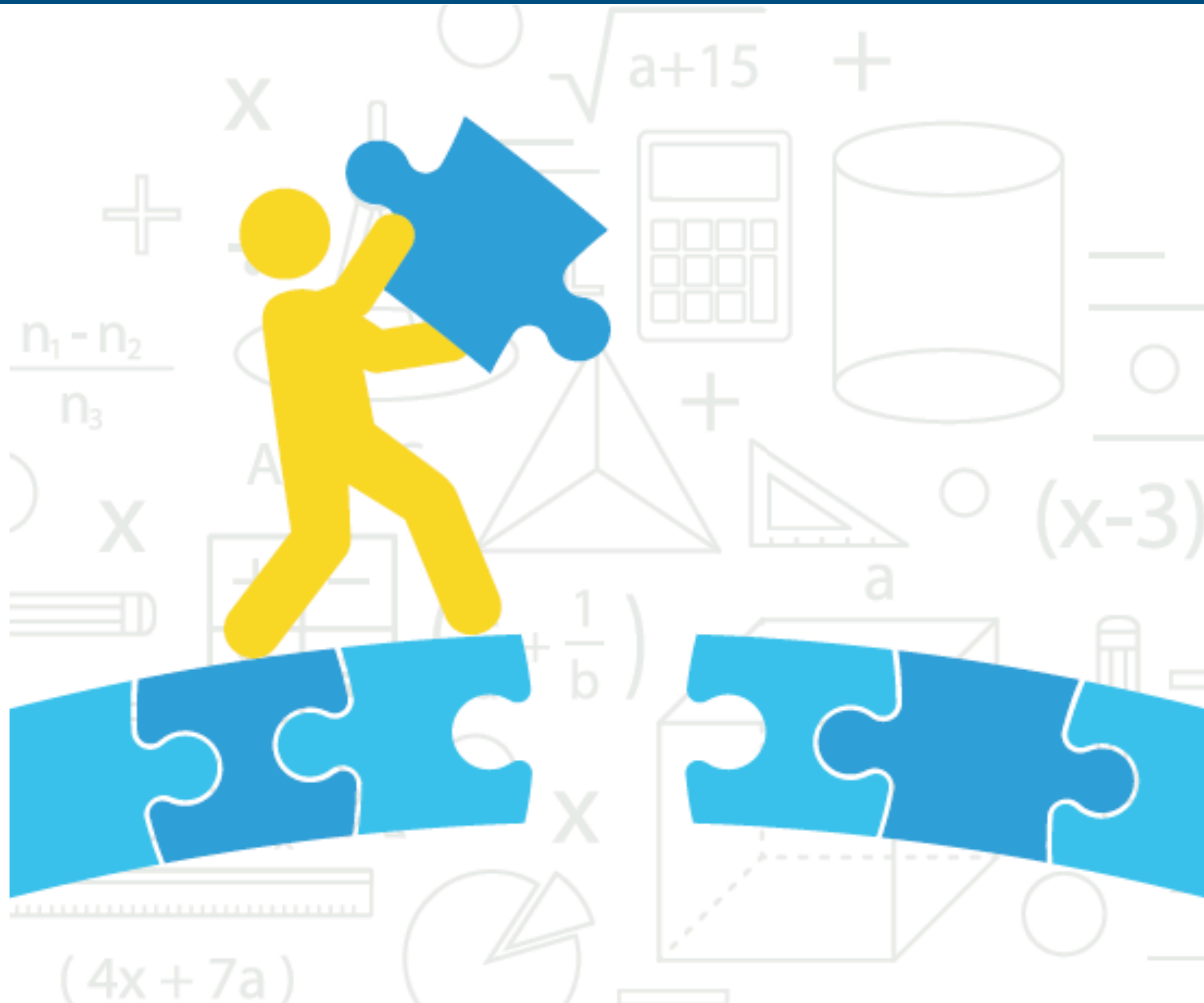
1 Match each sentence up to each algebraic expression.

"Multiply **b** by 2 then subtract from **a**."	$a(b - c)$
"Subtract **c** from **a** then multiply by **b**."	$\frac{a}{c} - b$
"Add **b** to **a** then divide into **c**."	$\frac{ab}{c}$
"Multiply **a** by **b** then divide by **c**."	$\frac{c}{a + b}$
"Divide **c** by **b** then multiply by **a**."	abc
"Multiply **a** by **c** then divide into **b**."	$b(a - c)$
"Add **b** to **a** then divide by **c**."	$a - 2b$
"Multiply **ab** by **c**."	$\frac{b}{ac}$
"Multiply **a** by 4 then subtract **c**."	$\frac{a + b}{c}$
"Subtract **c** from **b** then multiply by **a**."	$\frac{c}{b} \times a$

- Planning for retention: the spacing effect, interleaved practice

- Homework booklets: every child, every night policy

- Digitalisation of assessments



MATHEMATICAL TASKS

The Bridge Between Teaching and Learning

Chris McGrane and Mark McCourt

Designing Mathematical Tasks

A Practical Course with Chris McGrane

STARTING POINTS MATHS
TASKS AND BLOG

Workshop 1

- Why tasks matter
- Task design for fluency and understanding
- Spotting opportunities for task design
- Reflections on pre-course assignment
- Basic design principles

Workshop 2

- Mining rich activity from a routine task
- Adapting existing tasks
- Sequencing tasks for deeper understanding
- Task design to develop the mathematician
- Reflections on assignment 2

Workshop 3

- Variation Theory - beyond surface level differences
- Task design as a curriculum driver
- Task design from templates
- Spontaneous Task Design & ‘No Prep’ Task Design

Cost of course: £20

STARTING POINTS MATHS
TASKS AND BLOG

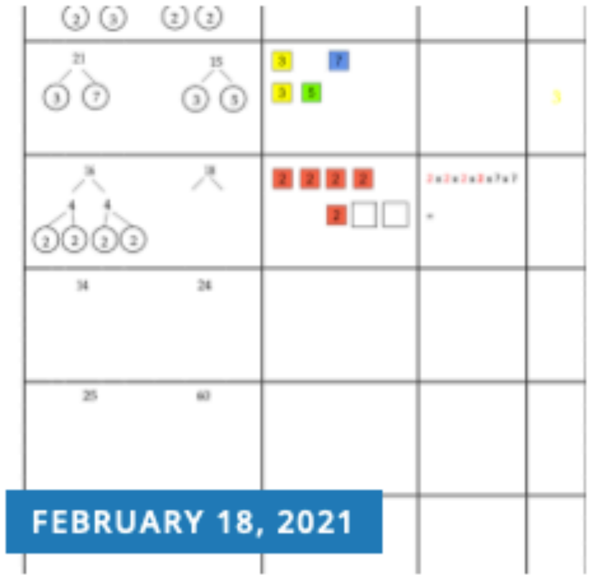
startingpointsmaths.com

@chrismcgrane84

Latest Tasks | Tasks by Topic | Higher Maths | Primary Maths | Task Design Book | Blog Articles | About

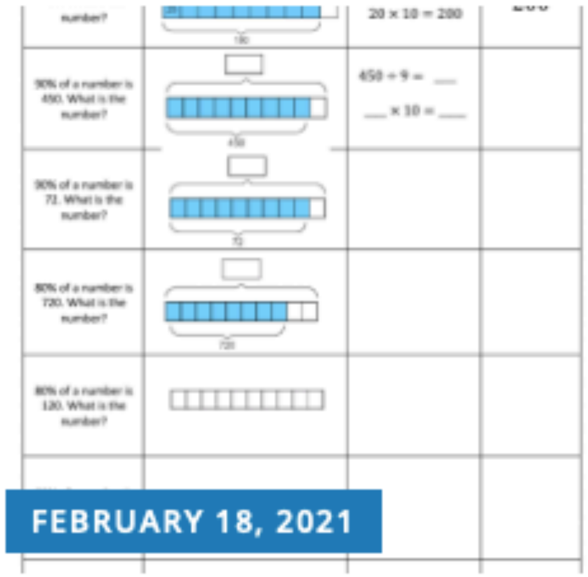
MARCH 2, 2021

Fraction Division Patterns



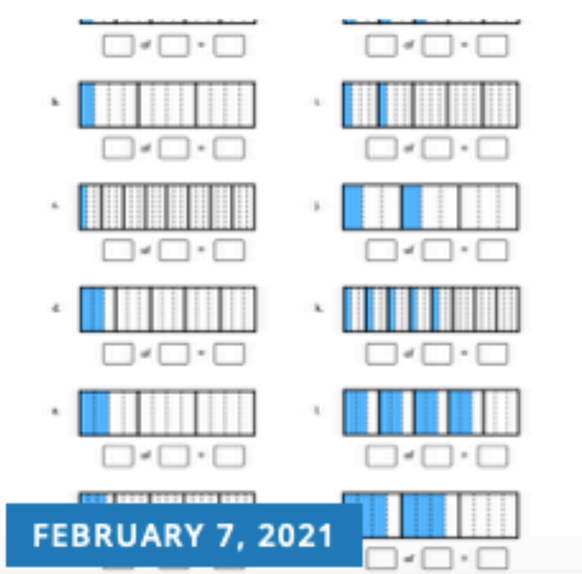
FEBRUARY 18, 2021

Prime Factorisation Using Prime Factor Tiles

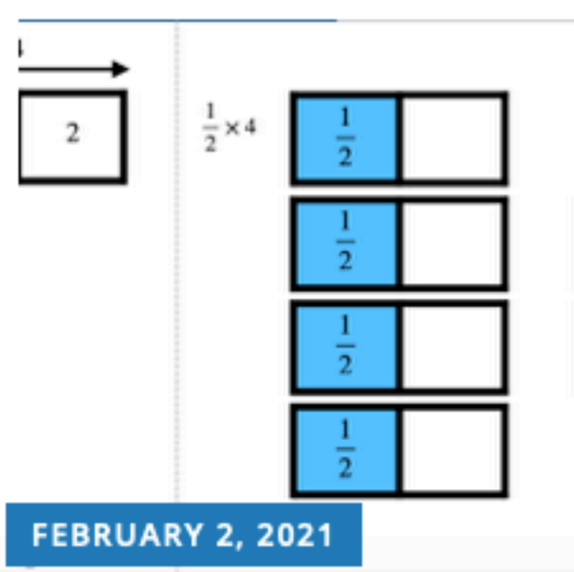


FEBRUARY 18, 2021

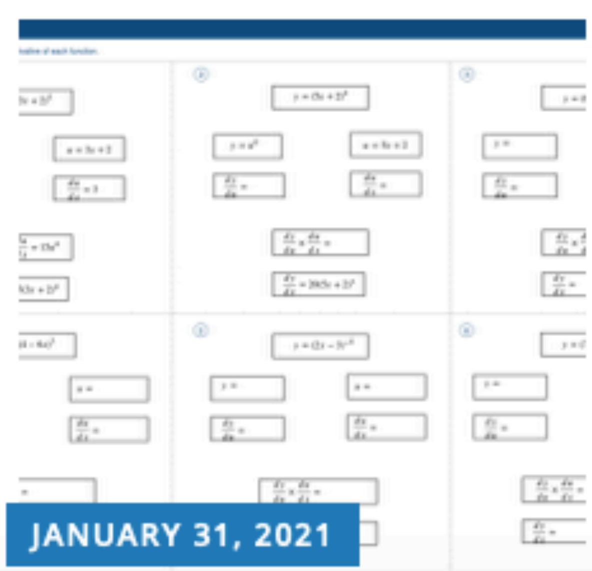
Reverse Percentages



FEBRUARY 7, 2021



FEBRUARY 2, 2021



JANUARY 31, 2021

Search Tasks and Blogs

Search …

Top Posts & Pages

- Order of Operations - Area Model
- Drawing altitudes, medians, perpendicular bisectors
- Straight Line - Everything we know!
- Evaluating Trig Expressions
- Functions Matching Activity
- Domain and Range
- Quadrants - Isolating Decisions
- m = tanθ sorting task
- Identifying information: