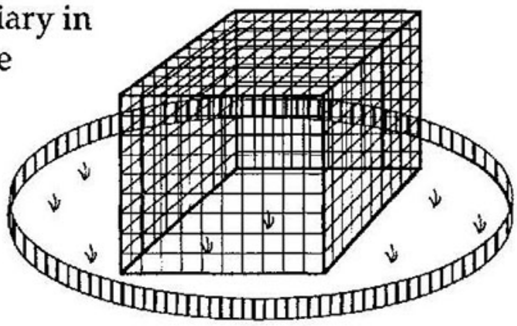


The owners of a zoo intend to build a new aviary in the shape of a cuboid with a square floor. The volume of the aviary will be 500 m^3.

(a) If x metres is the length of one edge of the floor, show that the area A square metres of netting required is given by

$$A = x^2 + \frac{2000}{x}.$$ **(4)**

(b) Find the dimensions of the aviary to ensure that the cost of netting is minimised. **(6)**

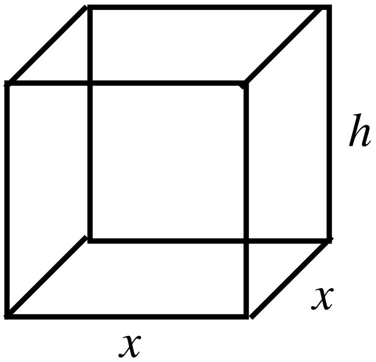
Part (a)

① Come up with a draft version of what they ask for.

$$A = front + back + left + right + top$$

$$A = xh + xh + xh + xh + x^2$$

$$A = x^2 + 4xh$$



We have an h, which the question doesn't!

② What other information is given?
Create an expression for this.

$$Volume = 500m^3 \qquad Volume = lbh$$

$$500 = lbh$$

$$500 = x^2h$$

$$h = \frac{500}{x^2}$$ Change the subject to h.

③ Substitute your expression from step 2 into the step 1 formula.

$$A = x^2 + 4xh \qquad h = \frac{500}{x^2}$$

$$A = x^2 + 4x\left(\frac{500}{x^2}\right)$$

$$A = x^2 + \frac{2000x}{x^2}$$

$$A = x^2 + \frac{2000}{x}$$

Part (b)

1 Find the stationary point(s)

$$A = x^2 + \frac{2000}{x}$$

$$A = x^2 + 2000x^{-1}$$ Prepare for differentiation

$$\frac{dA}{dx} = 2x - 2000x^{-2}$$ Differentiate

$$\frac{dA}{dx} = 2x - \frac{2000}{x^2}$$ Express without negative or fraction powers.

* For stationary points dA/dx = 0 *

$$2x - \frac{2000}{x^2} = 0$$

$$2x = \frac{2000}{x^2}$$

$$2x^3 = 2000$$

$$x^3 = 1000$$

$$x = \sqrt[3]{1000}$$

$$x = 10$$

x	→	10	→
$\frac{dA}{dx} = 2x - \frac{2000}{x^2}$	-	0	+
Slope	\	—	/

The nature table is essential!

X = 10 is a value which minimises the surface area.

$$h = \frac{500}{x^2}$$

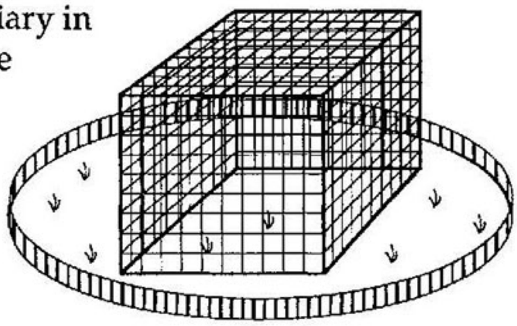
$$h = \frac{500}{10^2}$$

$$h = \frac{500}{100}$$

$$h = 5$$

Dimension are 10 by 10 by 5.

The owners of a zoo intend to build a new aviary in the shape of a cuboid with a square floor. The volume of the aviary will be 600 m^3.



(*a*) If x metres is the length of one edge of the floor, show that the area A square metres of netting required is given by

$$A = x^2 + \frac{2400}{x}.$$

(4)

(*b*) Find the dimensions of the aviary to ensure that the cost of netting is minimised.

(6)

Part (a)

1 Come up with a draft version of what they ask for.

2 What other information is given?
Create an expression for this.

3 Substitute your expression from step 2 into the step 1 formula.

Part (b)

(1) Find the stationary point(s)

Prepare for differentiation

Differentiate

Express without negative or fraction powers.

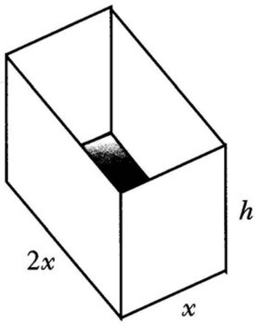
* For stationary points dA/dx = 0 *

The nature table is essential!

Dimension are:

1

An open cuboid measures internally x units by $2x$ units by h units and has an inner surface area of 12 units^2.

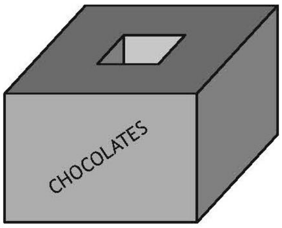


(*a*) Show that the volume, V units^3, of the cuboid is given by $V(x) = \frac{2}{3}x\left(6 - x^2\right)$. **3**

(*b*) Find the exact value of x for which this volume is a maximum. **5**

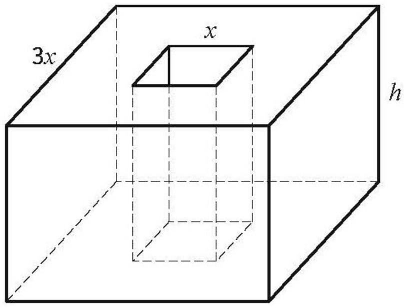
A manufacturer of chocolates is launching a new product in novelty shaped cardboard boxes.

2



The box is a cuboid with a cuboid shaped tunnel through it.

- The height of the box is h centimetres
- The top of the box is a square of side $3x$ centimetres
- The end of the tunnel is a square of side x centimetres
- The volume of the box is 2000 cm^3



(a) Show that the total surface area, A cm^2, of the box is given by

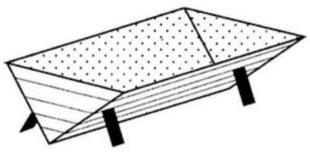
$$A = 16x^2 + \frac{4000}{x}.$$ **3**

(b) To minimise the cost of production, the surface area, A, of the box should be as small as possible.

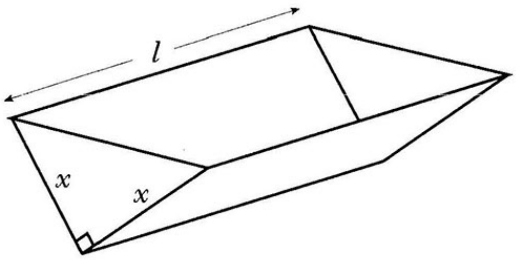
Find the minimum value of A. **6**

3

An open water tank, in the shape of a triangular prism, has a capacity of 108 litres. The tank is to be lined on the inside in order to make it watertight.



The triangular cross-section of the tank is right-angled and isosceles, with equal sides of length x cm. The tank has a length of l cm.



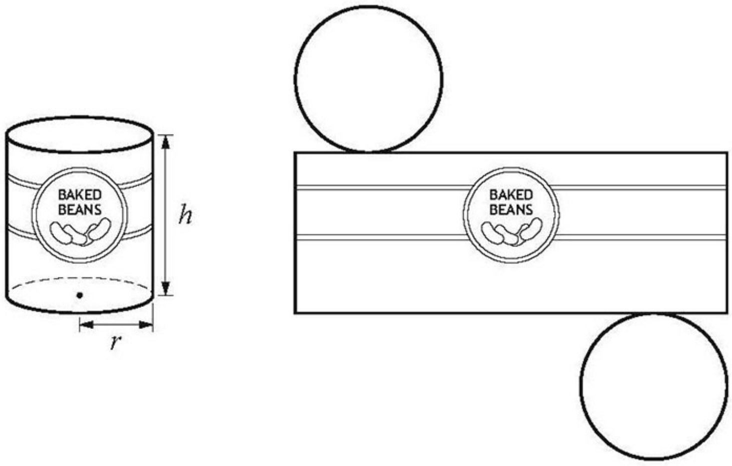
(*a*) Show that the surface area to be lined, A cm^2, is given by $A(x) = x^2 + \frac{432000}{x}$. **3**

(*b*) Find the value of x which minimises this surface area. **5**

A cylindrical tin of baked beans has a volume of 450 cm^3.

The radius of the tin is r cm and its height is h cm.

A net of the tin is shown in the diagram.



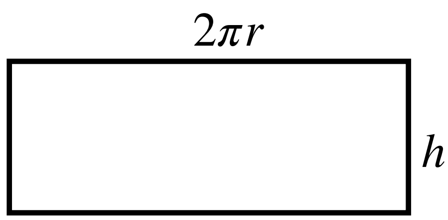
(a) Show that the surface area of the tin, A square centimetres, is given by

$$A(r) = 2\pi r^2 + \frac{900}{r}.$$ 3

(b) Determine the radius that will minimise the surface area. 6

Part (a)

1 Come up with a draft version of what they ask for.



$$A(r) = top + bottom + sides$$

$$A(r) = \pi r^2 + \pi r^2 + 2\pi r h$$

$$A(r) = 2\pi r^2 + 2\pi r h$$

We have an h, which the question doesn't!

2 What other information is given?
Create an expression for this.

$$Volume = 450m^3$$

$$Volume = area\ of\ base \times height$$

$$Volume = \pi r^2 h$$

$$450 = \pi r^2 h$$

$$h = \frac{450}{\pi r^2}$$

Change the subject to h.

3 Substitute your expression from step 2 into the step 1 formula.

$$A(r) = 2\pi r^2 + 2\pi r h \qquad h = \frac{450}{\pi r^2}$$

$$A(r) = 2\pi r^2 + 2\pi r\left(\frac{450}{\pi r^2}\right)$$

$$A(r) = 2\pi r^2 + \frac{900\pi r}{\pi r^2}$$

$$A(r) = 2\pi r^2 + \frac{900}{r}$$

Part (b)

1 Find the stationary point(s)

$$A(r) = 2\pi r^2 + \frac{900}{r}$$

$A(r) = 2\pi r^2 + 900r^{-1}$ Prepare for differentiation

$A'(r) = 4\pi r - 900r^{-2}$ Differentiate

$A'(r) = 4\pi r - \frac{900}{r^2}$ Express without negative or fraction powers.

* For stationary points A′(r) = 0 *

$$4\pi r - \frac{900}{r^2} = 0$$

$$4\pi r = \frac{900}{r^2}$$

$$4\pi r^3 = 900$$

$$r^3 = \frac{900}{4\pi}$$

$$r = \sqrt[3]{\frac{900}{4\pi}}$$

$$r = 4.15$$

x	→	4.15	→
$A'(r) = 4\pi r - \frac{900}{r^2}$	-	0	+
Slope	\	—	/

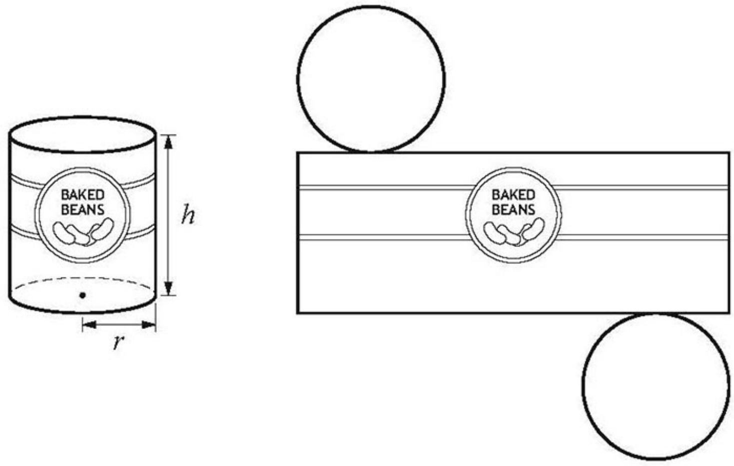
The nature table is essential!

r = 4.15 is a value which minimises the surface area.

A cylindrical tin of baked beans has a volume of 500 cm^3.

The radius of the tin is r cm and its height is h cm.

A net of the tin is shown in the diagram.



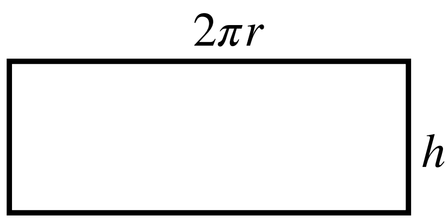
(a) Show that the surface area of the tin, A square centimetres, is given by

$$A(r) = 2\pi r^2 + \frac{1000}{r}.$$ **3**

(b) Determine the radius that will minimise the surface area. **6**

Part (a)

1 Come up with a draft version of what they ask for.



We have an h, which the question doesn't!

2 What other information is given?
Create an expression for this.

Change the subject to h.

3 Substitute your expression from step 2 into the step 1 formula.

Part (b)

(1) Find the stationary point(s)

Prepare for differentiation

Differentiate

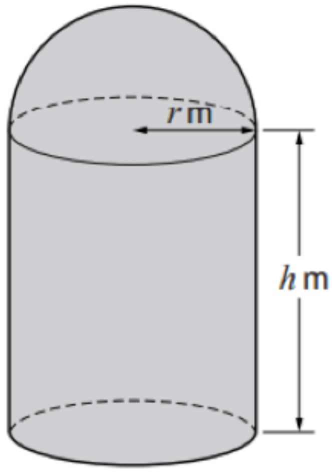
Express without negative or fraction powers.

* For stationary points A′(r) = 0 *

The nature table is essential!

1. A design for a new grain container is in the shape of a cylinder with a hemispherical roof and a flat circular base. The radius of the cylinder is r metres, and the height is h metres.

The volume of the **cylindrical** part of the container needs to be 100 cubic metres.

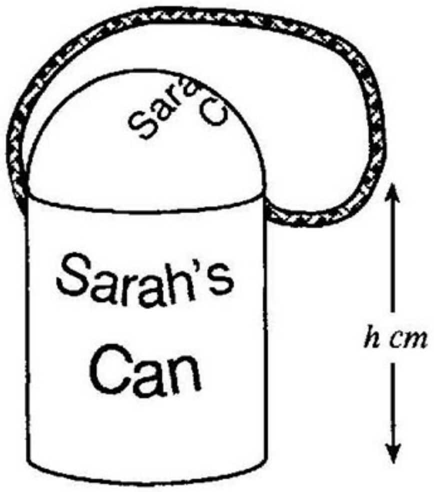


(a) Given that the curved surface area of a hemisphere of radius r is $2\pi r^2$ show that the surface area of metal needed to build the grain container is given by:

$$A = \frac{200}{r} + 3\pi r^2 \text{ square metres}$$

(b) Determine the value of r which minimises the amount of metal needed to build the container.

2. A child's drinking beaker is in the shape of a cylinder with a hemispherical lid and a circular flat base. The radius of the cylinder is r cm and the height is h cm. The volume of the cylinder is 400 cm^3.



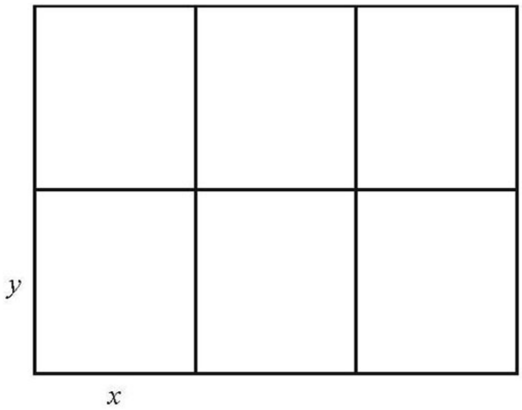
(*a*) Show that the surface area of plastic, $A(r)$, needed to make the beaker is given by $A(r) = 3\pi r^2 + \frac{800}{r}$. **(3)**

Note: The curved surface area of a hemisphere of radius r is $2\pi r^2$.

(*b*) Find the value of r which ensures that the surface area of plastic is minimised. **(6)**

1 A council is setting aside an area of land to create six fenced plots where local residents can grow their own food.

Each plot will be a rectangle measuring x metres by y metres as shown in the diagram.



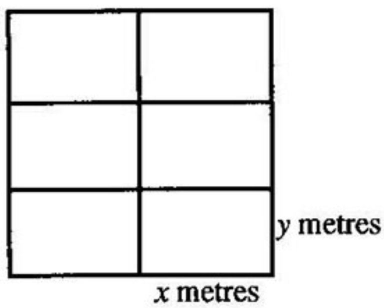
(a) The area of land being set aside is $108\ \text{m}^2$.

Show that the total length of fencing, L metres, is given by

$$L(x) = 9x + \frac{144}{x}.$$ 3

(b) Find the value of x that minimises the length of fencing required. 6

2 A zookeeper wants to fence off six individual animal pens.



Each pen is a rectangle measuring x metres by y metres, as shown in the diagram.

(*a*) (i) Express the total length of fencing in terms of x and y.

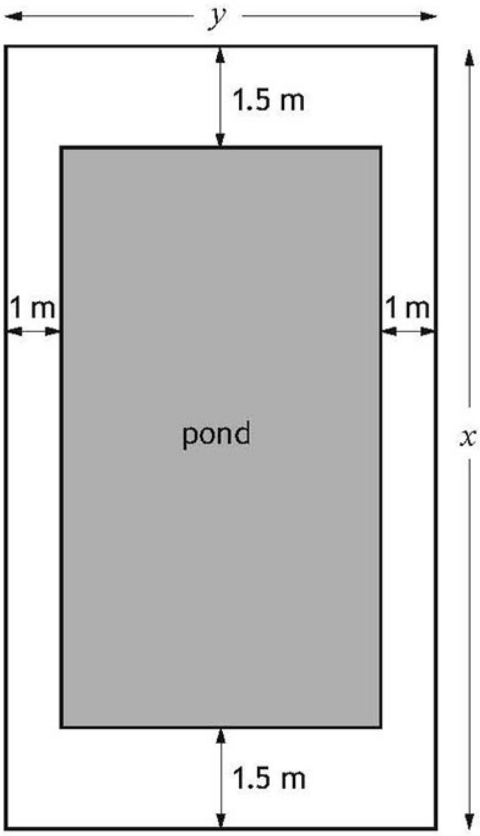
(ii) Given that the total length of fencing is 360m, show that the total area, $A\ \text{m}^2$, of the six pens is given by $A(x) = 240x - \frac{16}{3}x^2$. 4

(*b*) Find the values of x and y which give the maximum area and write down this maximum area. 6

3 A rectangular plot consists of a rectangular pond surrounded by a path.

The length and breadth of the plot are x metres and y metres respectively.

The path is 1.5 metres wide at the ends of the pond and 1 metre wide along the other sides as shown.



The total area of the **pond and path together** is 150 square metres.

(a) Show that the area of the pond, A square metres, is given by

$$A(x) = 156 - 2x - \frac{450}{x}.$$ **3**

(b) Determine the maximum area of the pond. **6**